

# Modélisation de la densité électronique : apport des rayonnements synchrotron et neutronique et applications

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# OUTLINE

- **Structure factor and electron density analysis**  
**Multipole Model , topological analysis and electrostatics**
- **Applications to**
  - Quantum Chemistry :Chemical Bonding**
  - Phase transitions :TTF CA**
  - Metastable states :Thermal and photoexcited molecules**
  - Material Science : Electrostatics in zeolite**
- **Problems with heavy elements and need of SR**

# What do we measure?

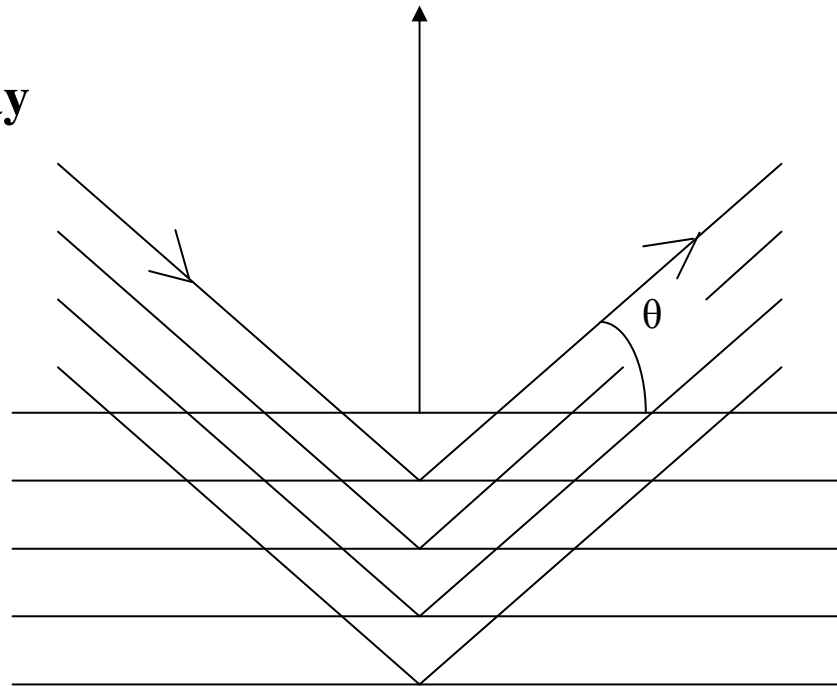
*X-ray case*

Laboratory or Synchrotron Source

$\vec{H}$

**X-ray**

Point or **CCD detector**



$$\Rightarrow I(\vec{H}) = \Gamma^2(\vec{H}) * F^2(\vec{H})$$

**Interference function**

Fourier: transform of the Crystal Lattice

**Structure factor**

Fourier transform of the Thermally Smeared Electron Density

# Charge density refinement

$$I_{\text{Bragg}} = K \cdot \Gamma^2 \cdot F^2$$

$F^2$  : **Structural Informations**

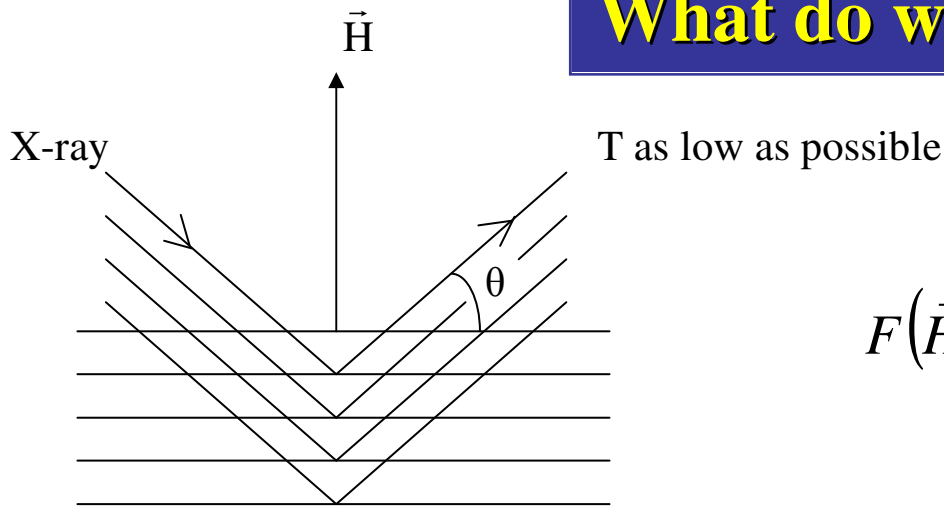
Accurate structure of metastables states

Anisotropic displacement parameters

Valence electron density

depends on  $\sin(\theta) / \lambda$  resolution

## What do we measure?



$$F(\vec{H}) = \int_{\text{maille}} \rho(\vec{r}) \exp(2i\pi\vec{H} \cdot \vec{r}) d^3\vec{r}$$

What we 'see' is the **Dynamic Electron Density** which is therefore the Inverse Fourier Transform of the Dynamic Structure Factors

$$\rho(\vec{r}) = \rho(\vec{r})_{\text{stat}} \otimes P(u) \quad \text{Atom probability density function}$$

Convolution theorem

$$F(\vec{H}) = F(\vec{H})_{\text{stat}} T(\vec{H})_{\text{DebyeWaller}}$$

$$F_{\text{stat}}(\vec{H}) = \sum_{j=1}^{\text{Na}} f_j(\vec{H}) \exp(2i\pi\vec{H} \cdot \vec{r}_j)$$

$f_j$  : scattering factor : Fourier transform of the electron density of atom  $j$

# Crystal structure refinement: X, Y, Z, U<sup>ij</sup>

$$\rho^{pro}(r) = \sum_{j=1}^{Na} \rho_j^{at}(r - r_j)$$

promolecule density                      free atom electron density

As, due to chemical bonding and interatomic interactions,  
the valence density is aspherical



Bias in x,y,z, U<sup>ij</sup>

Therefore:

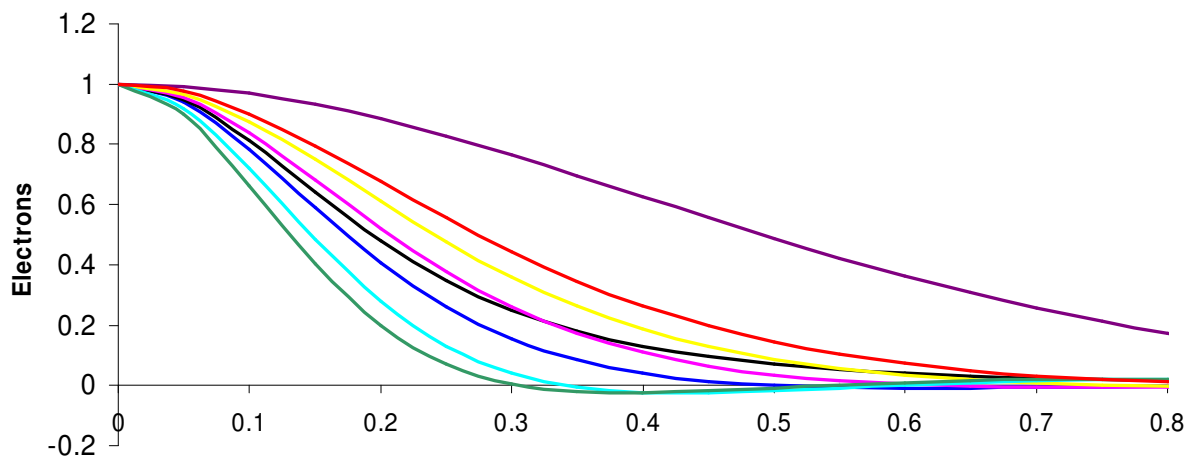
X-n                      xyz, U<sup>ij</sup> of all atoms including H refined from  
neutron data

X-X                      xyz, U<sup>ij</sup>                      from HO data but what  
about H atoms ?

The aspherical atom refinement must start with these  
positional and thermal parameters

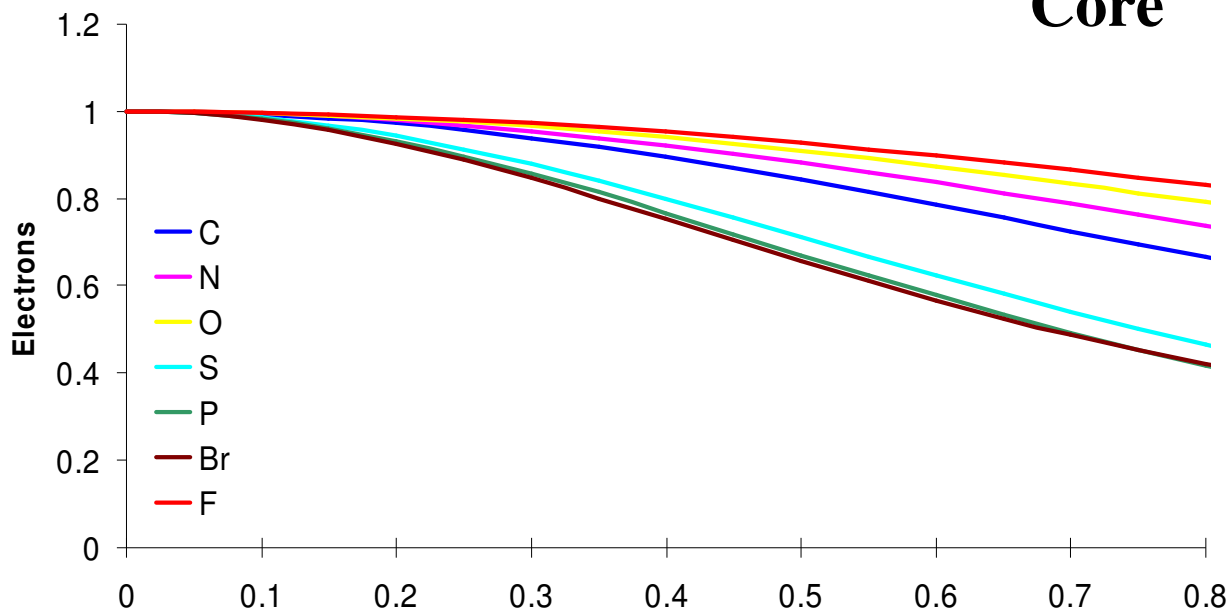
# Scattering factors normalized at 1 e<sup>-</sup>

Valence



$\text{\AA}^{-1}$

Core



$\text{\AA}^{-1}$

## X-n and HO refinement

- **X-n method** (Coppens Science, 158, 1577 (1967))

$$F_n(\vec{H}) = \sum_{j=1}^{Nat} b_j \exp 2\pi i \vec{H} \cdot \vec{r} T_j(\vec{H})$$

(x,y,z) and thermal (Uij)

**High order refinement (H.O)** uses FT properties

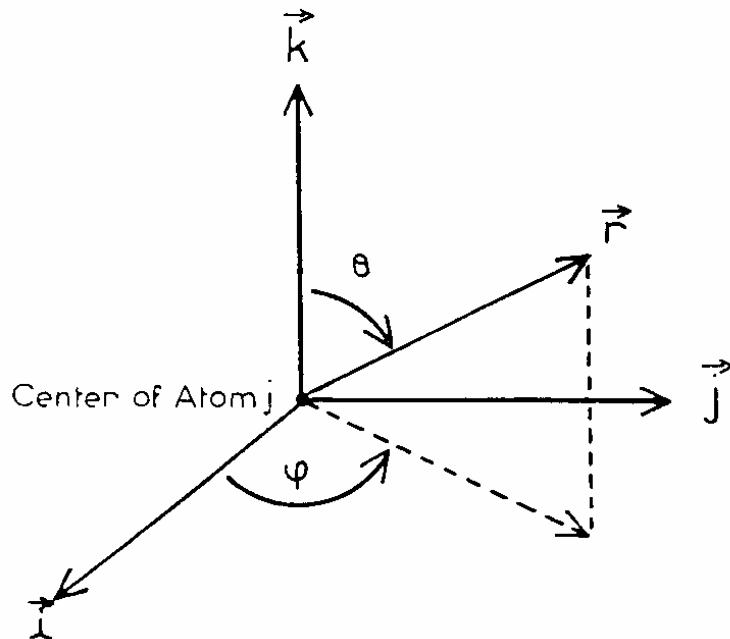
$$f(\vec{H}) = \int_V \rho(\vec{r}) \exp 2\pi i \vec{H} \cdot \vec{r} d^3 \vec{r}$$



# Main ideas and advantages of the aspherical electron density model

$$\rho^{at}(\vec{r}) = \rho_1^{at}(\vec{r}) + \delta\rho^{at}(\vec{r})$$

$$\delta\rho^{at}(\vec{r}) = \sum C_n R_n(r) A(\theta, \varphi)$$



*Atomic Orthogonal Frame  
Allows to take into account  
the atomic local symmetry*

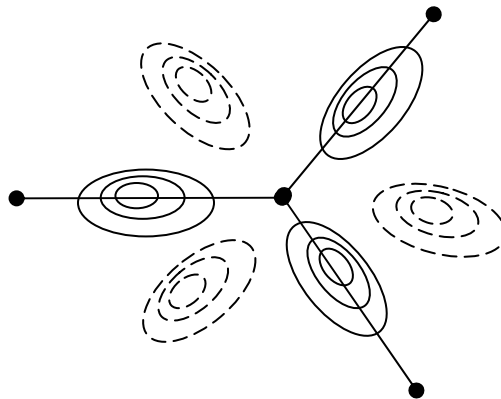
- . Symmetry restrictions of the multipole have been given by Kurki-Suonio (Israel J. of Chemistry, **16**, 115-123, 1977).

## An example $sp^2$ carbon atom

Electron density difference map

$$\Delta\rho_{\text{exp}} = FT^{-1} (F_o - F_{c_{\text{sph}}})$$

$sp^2$  carbon



Deformation density

$$\Delta\rho_{\text{exp}}(\vec{r})$$

→ Radial functions: maximas at the middle of the bonds

→ Angular functions: 3 fold symmetry

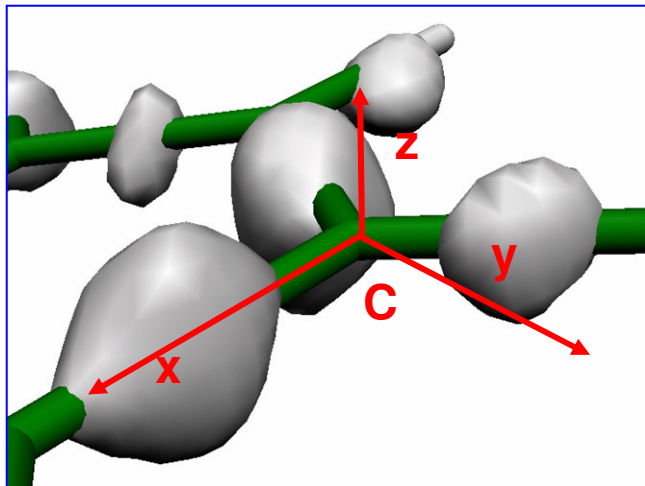
$Y_3^m$  (Stewart 1976) or  $\text{trig cos}^3\theta$  (Hirshfeld 1975)

HANSEN COPPENS MULTIPOLE MODEL coded in Molly and MOPRO programs

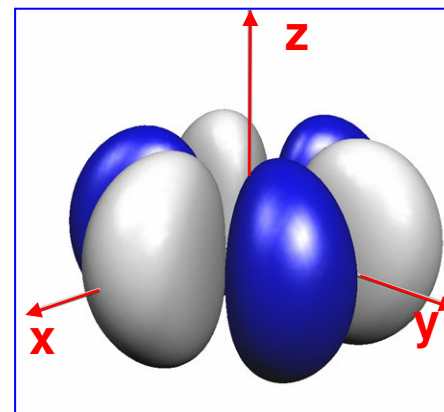
$$\rho(\mathbf{r}) = \rho_c^s(\mathbf{r}) + \kappa^3 \mathbf{P}_v \rho_v^s(\kappa \cdot \mathbf{r}) + \sum_{l=0, l_{max}} \kappa'^3 R_l(\kappa' \cdot \mathbf{r}) \sum_{m=\pm l} \mathbf{P}_{lm} Y_{lm}(\theta, \phi)$$

$$R_l(r) = \frac{\xi_l^{n_l+3}}{(n_l+2)!} r^{n_l} e^{-\xi_l \cdot r}$$

Atomic local frame  
x y z



use of local symmetry



Octupolar real  
spherical harmonic  
( $l;m$ )=(3;+3)

Hansen & Coppens (1978).  
*Acta Cryst.* A34, 909-921.

Jelsch, Guillot, Lagoutte and Lecomte 2004  
*J. Appl. Cryst.*, 38, 38-54.

# Topological analysis

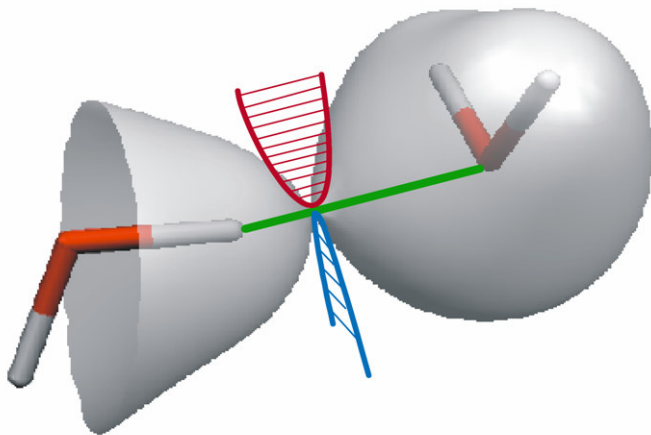
Critical points  $\vec{\nabla}\rho = \vec{0}$

Hessian matrix  $H_{ij} = \frac{\partial^2 \rho}{\partial x_i \partial x_j}$  Diagonalisation to get eigenvalues (curvatures  $\lambda_i$  and eigenvectors)

Classification  $W$ : number of non zero eigenvalues  
 CP( $W, \sigma$ )  $\sigma$ :  $\Sigma$  signs = signature

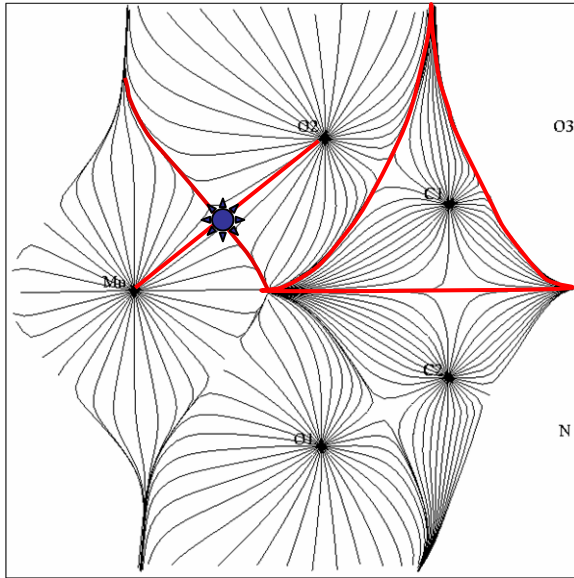
CP

3,-3 attractor:  
 3,+3 cage  
 3,+1 cycle  
 3,-1 interaction



$\lambda_1, \lambda_2 < 0 \implies$  maximum  
 $\lambda_3 > 0 \implies$  minimum

# Topological analysis



- Gradient trajectories  $\vec{\nabla}\rho(\vec{r}) = \frac{\partial\rho(\vec{r})}{\partial x} \vec{i} + \frac{\partial\rho(\vec{r})}{\partial y} \vec{j} + \frac{\partial\rho(\vec{r})}{\partial z} \vec{k}$
- Laplacien  $\nabla^2 = \sum_{i=1}^3 \frac{\partial^2 \rho}{\partial x_i^2}$   $\nabla^2 > 0$   $E_c$ , dilution  
 $\nabla^2 < 0$   $E_p$ , concentration
- Interatomic Surface  $\vec{\nabla}\rho(\vec{r}) \cdot \vec{n}(\vec{r}) = 0$
- Bond Path: direct interactions

## Topological integration: charges and atomic volumes

Interatomic surfaces define atomic basins

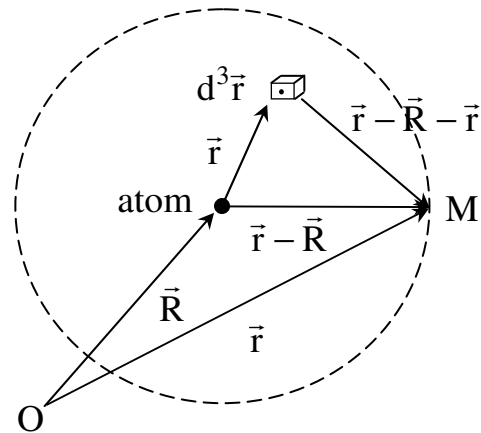
Integration on atomic basins (NEWPROP, Souhassou, Nancy) defines

-Atomic Volume and related electronic properties

Charge : 
$$q(V) = Z - \int_V \rho(\vec{r}) d^3\vec{r}$$

# Electrostatic potential calculation    Electros ( Ghermani , Lecomte et Al)

From multipole refinement (Stewart , 1976 )



$$V(\mathbf{r}) = V_{\text{core}}(\mathbf{r}) + V_{\text{val}}(\mathbf{r}) + \Delta V(\mathbf{r})$$

with

$$V_{\text{core}}(\mathbf{r}) = \frac{Z}{|\mathbf{r} - \mathbf{R}|} - \int \frac{\rho_{\text{core}}(\mathbf{r}')}{|\mathbf{r} - \mathbf{R} - \mathbf{r}'|} d^3\mathbf{r}'$$

and

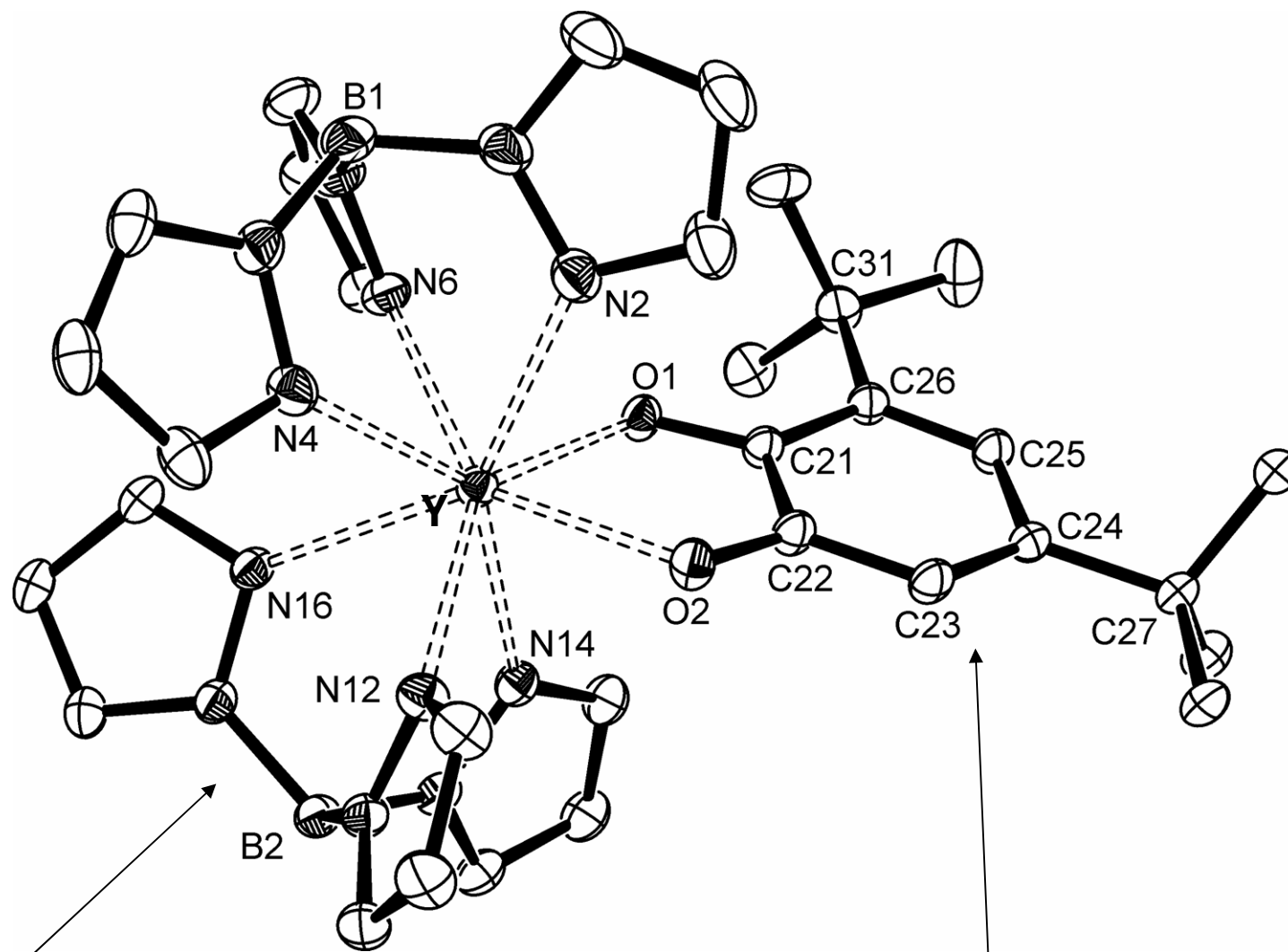
$$V_{\text{val}}(\mathbf{r}) = - \int P_{\text{val}} / \kappa'^3 \frac{\rho_{\text{val}}(\kappa' \mathbf{r}')}{|\mathbf{r} - \mathbf{R} - \mathbf{r}'|} d^3\mathbf{r}'$$

$$\Delta V(\mathbf{r}) = 4\pi \sum_{lm} \frac{\kappa'' P_{lm}}{2l+1} \left[ \frac{1}{\kappa''^{l+1} |\mathbf{r} - \mathbf{R}|^{l+1}} \int_0^{\kappa'' |\mathbf{r} - \mathbf{R}|} t^{l+2} R_{nl}(t) dt + \kappa''^l |\mathbf{r} - \mathbf{R}|^l \int_{\kappa'' |\mathbf{r} - \mathbf{R}|}^{\infty} \frac{1}{t^{l-1}} R_{nl}(t) dt \right] y_{lm} \pm (\theta', \phi')$$

# OUTLINE

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# I) Experimental Charge density and topology of the B-N bond



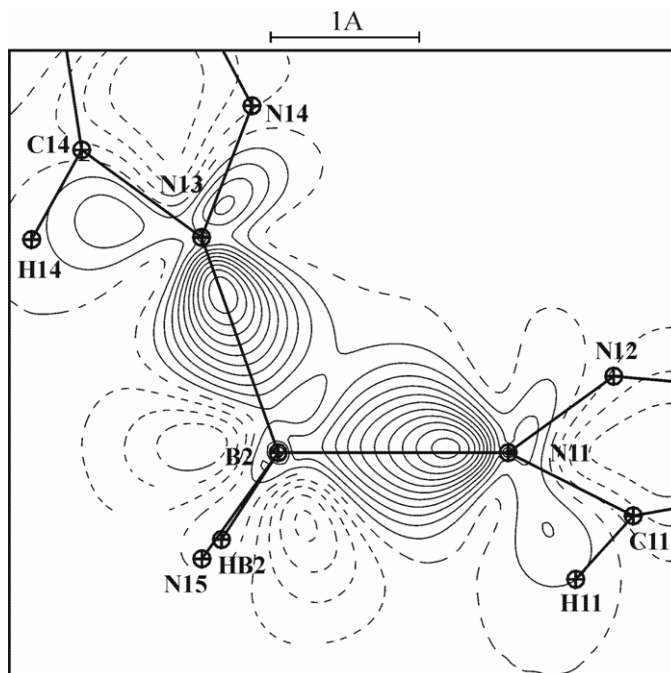
hydrotrispyrazolylborate

semiquinone



# EXPERIMENTAL STATIC DEFORMATION DENSITY

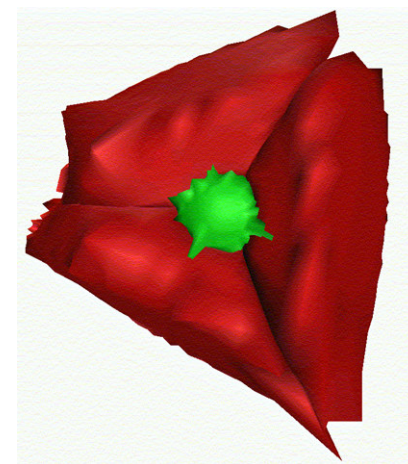
## B-N deformation density



*Polarisation of the electron density towards the N atom : this observation may be quantified by the topological analysis of rho*

# Topological properties of the B-N and B-H bonds

Atom	B	N	H
Topological charge +	2.54	-0.94	-0.61
Atomic volume (A <sup>3</sup> )	0.83	12.82	13.4



$$q(V) = Z - \int \rho(\vec{r}) d^3 \vec{r}$$

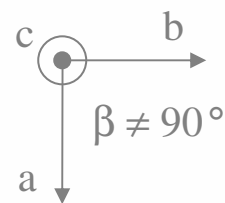
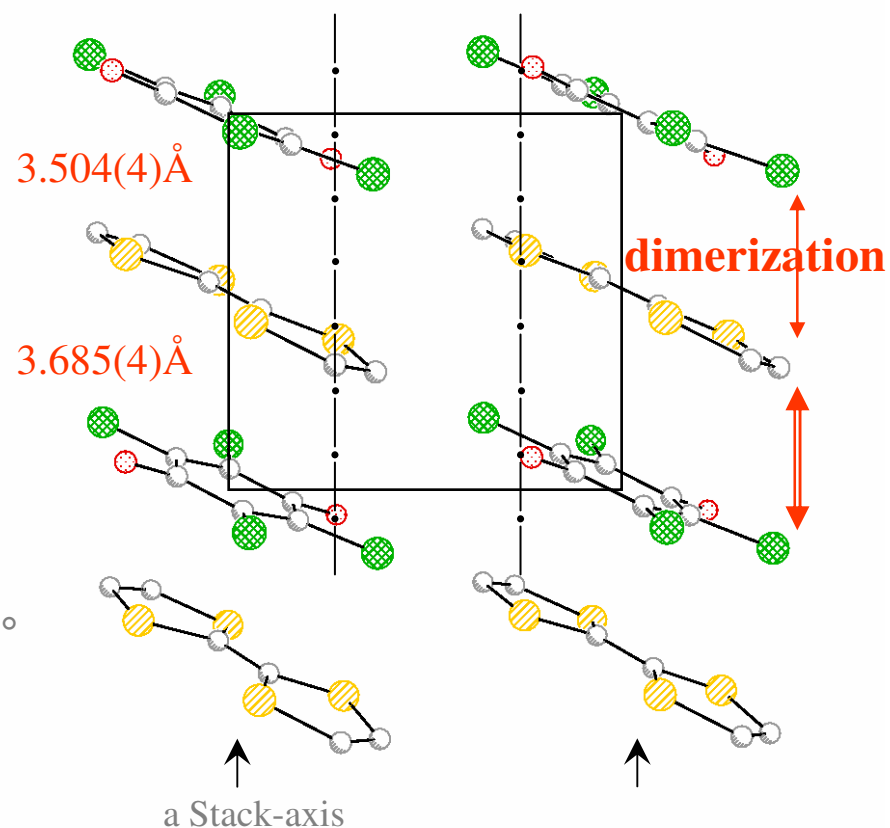
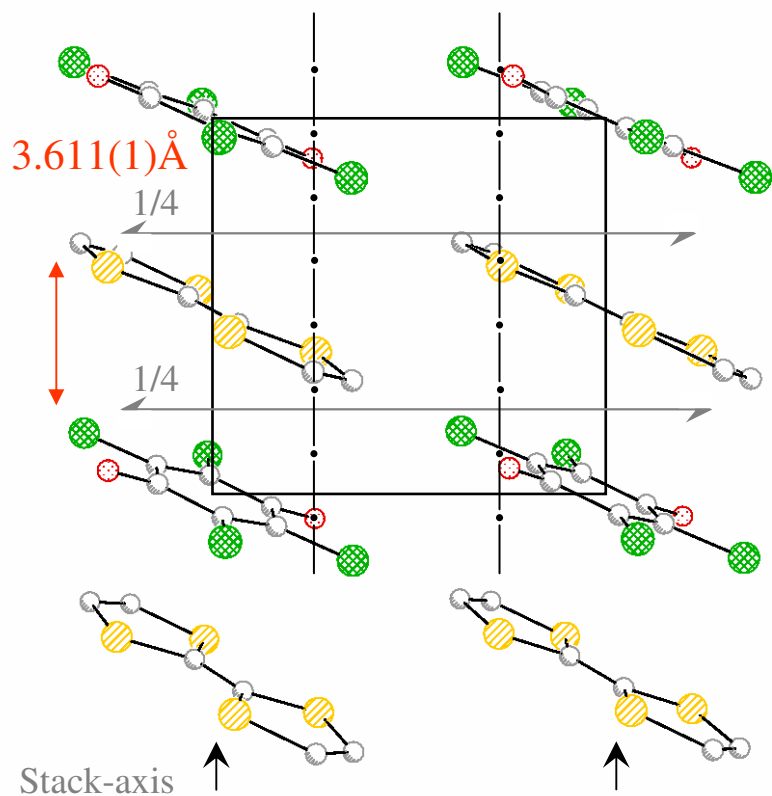
# Neutral phase $P2_1/n$

# TTF CA

# Ionic phase $Pn$

$a = 7.230(1) \text{ \AA}$ ,  $b = 7.595(1) \text{ \AA}$ ,  $c = 14.499(1) \text{ \AA}$ ,  $\beta = 99.1(1)^\circ$

$a = 7.191(1) \text{ \AA}$ ,  $b = 7.540(1) \text{ \AA}$ ,  $c = 14.441(1) \text{ \AA}$ ,  $\beta = 98.6(1)^\circ$



... D<sup>+</sup>A<sup>-</sup>D<sup>+</sup>A<sup>-</sup>D<sup>+</sup>A<sup>-</sup>D<sup>+</sup>A<sup>-</sup>D<sup>+</sup>A<sup>-</sup>D<sup>+</sup>A<sup>-</sup> ...

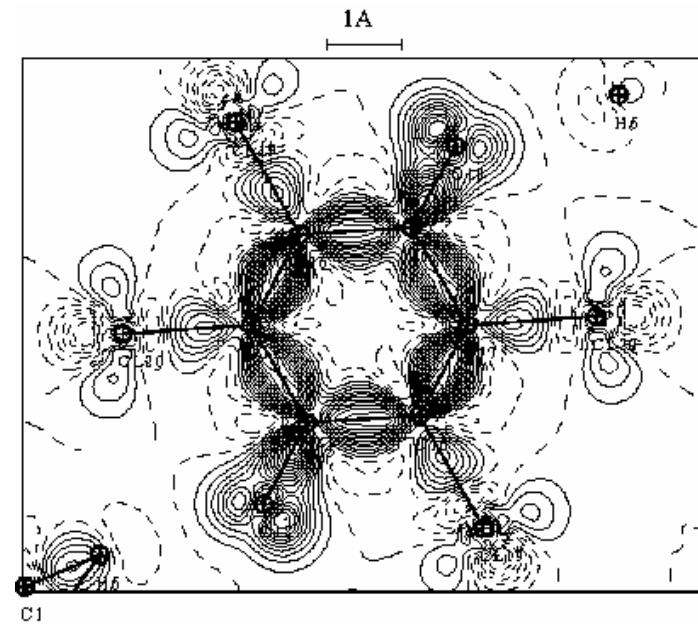
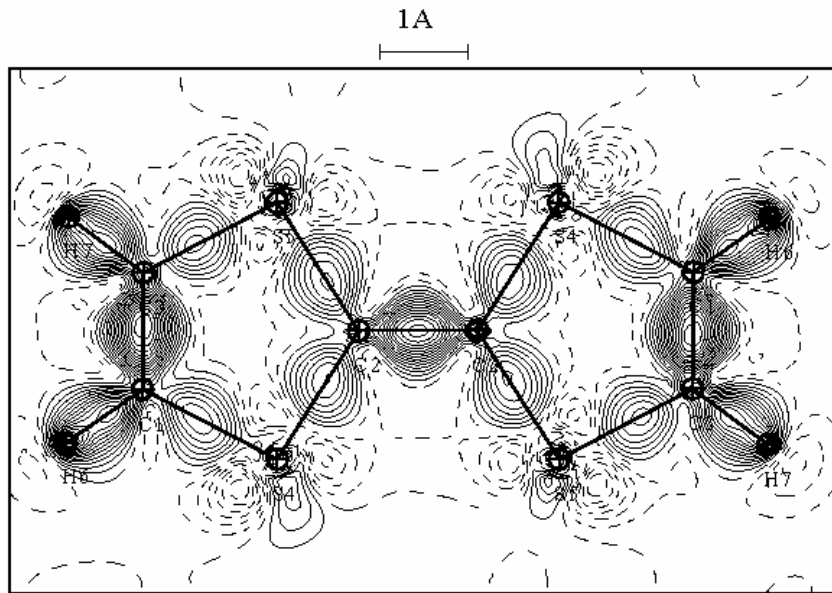
$T_{N-I} = 81 \text{ K}$

... (D<sup>+</sup>A<sup>-</sup>) (D<sup>+</sup>A<sup>-</sup>) (D<sup>+</sup>A<sup>-</sup>) (D<sup>+</sup>A<sup>-</sup>) (D<sup>+</sup>A<sup>-</sup>) (D<sup>+</sup>A<sup>-</sup>) ...

# Crystallographic Experiments : TTF-CA

	TTF-CA	TTF-CA
Formula	$C_{12}S_4Cl_4O_2H_4$	$C_{12}S_4Cl_4O_2H_4$
M	900.38	900.38
T(K)	105	15
Crystal System	Monoclinic	Monoclinic
Space Group	$P2_1/n$	$Pn$
Cell Parameters	a= 7.2297(5) Å b= 7.5933(3) Å c=14.4980(9) Å $\beta=99.15(3)^\circ$	a= 7.1999(9) Å b= 7.5556(6) Å c=14.479(1) Å $\beta=98.511(8)^\circ$
Volume (Å <sup>3</sup> )	786.2 (2)	779.0(5)
Z	2	2
Réfl. Measured / uniques	168470 / 9237	75177 / 14634
$\mu$ (Mo-K $\alpha$ ) (mm <sup>-1</sup> ), Résolution (Å <sup>-1</sup> )	1.28, 1.14	1.29, 1.16
Internal agreement $R_{int}$	0.0364	0.0266
R (I>3 $\sigma$ (I)) <i>End of multipole refinement</i>	0.0139	0.0124

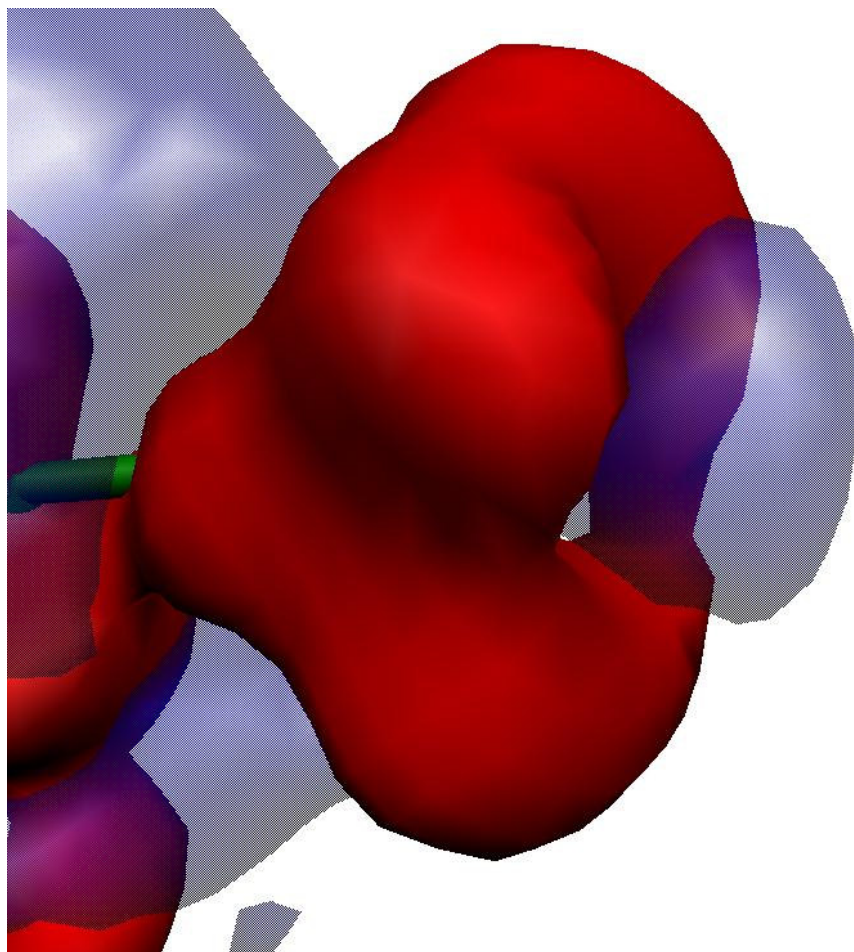
# Neutral Phase: Static Déformation Density



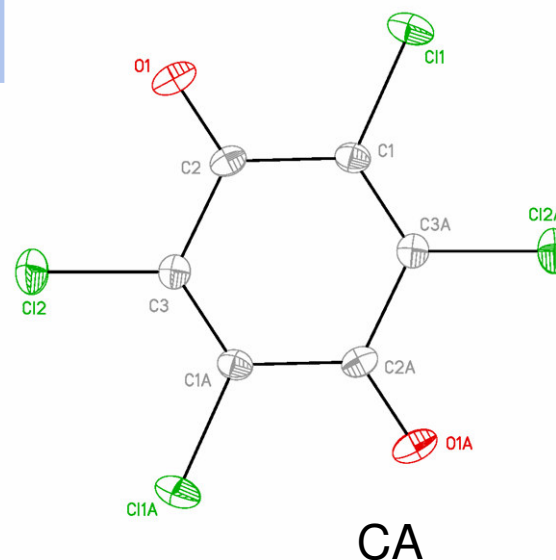
Contours,  $0.05 \text{ e}\text{\AA}^{-3}$

$$\Delta\rho_{stat}(\mathbf{r}) = \sum_{j=1}^{N_{atoms}} \left\{ \left[ P_v \kappa^3 \rho_v(\kappa\mathbf{r}) - N_v \rho_v(\mathbf{r}) \right] + \sum_{l=0}^{l_{max}} \kappa'^3 R_l(\kappa'\mathbf{r}) \sum_{m=-l}^l P_{lm} y_{lm}(\theta, \varphi) \right\}_j$$

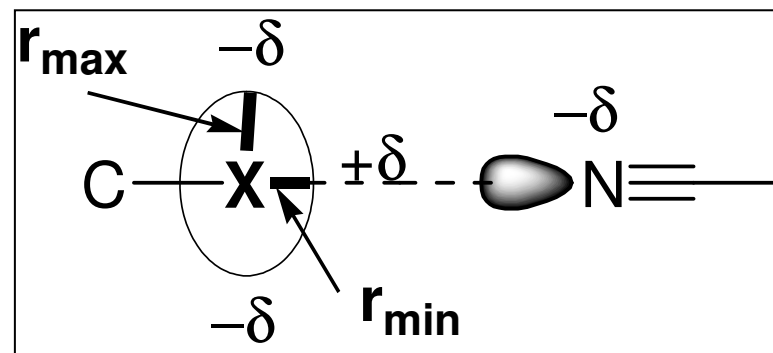
## Cl Deformation density



Isocontour =  $0.05 \text{ e}/\text{\AA}^3$

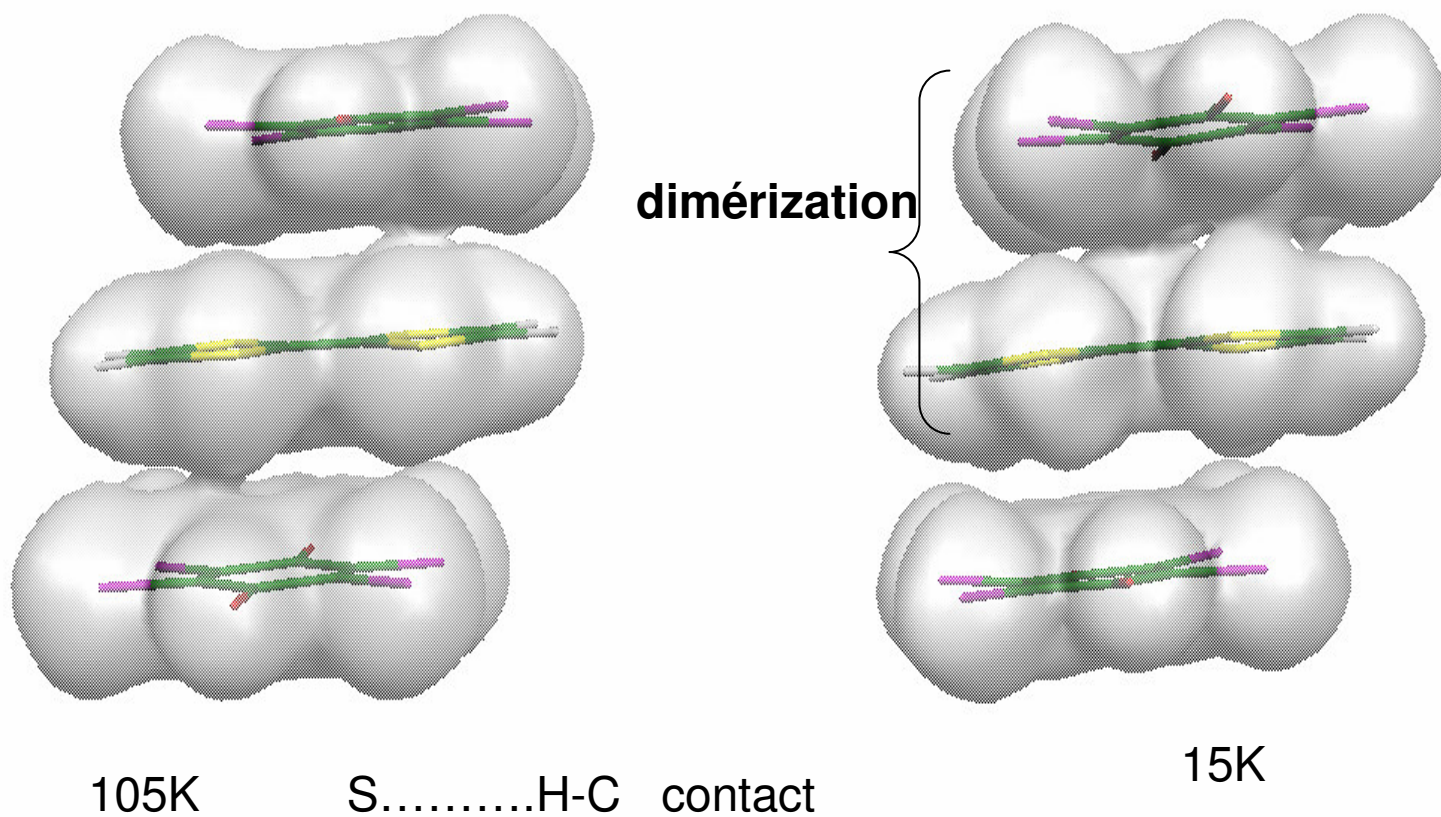


Anisotropy of the Cl charge density  
And intermolecular interactions  
As proposed to understand halogen bonds





# Total Charge density of TTF-CA in both N and I phases



Isodensity =  $0.065 \text{ e}/\text{\AA}^3$

# Direct Estimation of the charge transfer by X ray diffraction

$$: \quad \rho_{atom}(\vec{r}) = \rho_{core}(r) + P_v \kappa^3 \rho_v(\kappa r)$$

$$\rho_{atom}(\vec{r}) = \rho_{core}(r) + P_v \kappa^3 \rho_v(\kappa r) + \sum_{l=0}^{l_{max}} \kappa'^3 R_l(\kappa' r) \sum_{m=-l}^l P_{lm} y_{lm}(\theta, \varphi)$$

⇒ Estimated atomic Charge:  $q_{atom} = N_{atom} - P_v$

OR

Topological Analysis : Integration in atomic basins (Bader)

$$\Rightarrow \quad \vec{\nabla} \rho(\vec{r}) \cdot \vec{n}(\vec{r}) = 0$$

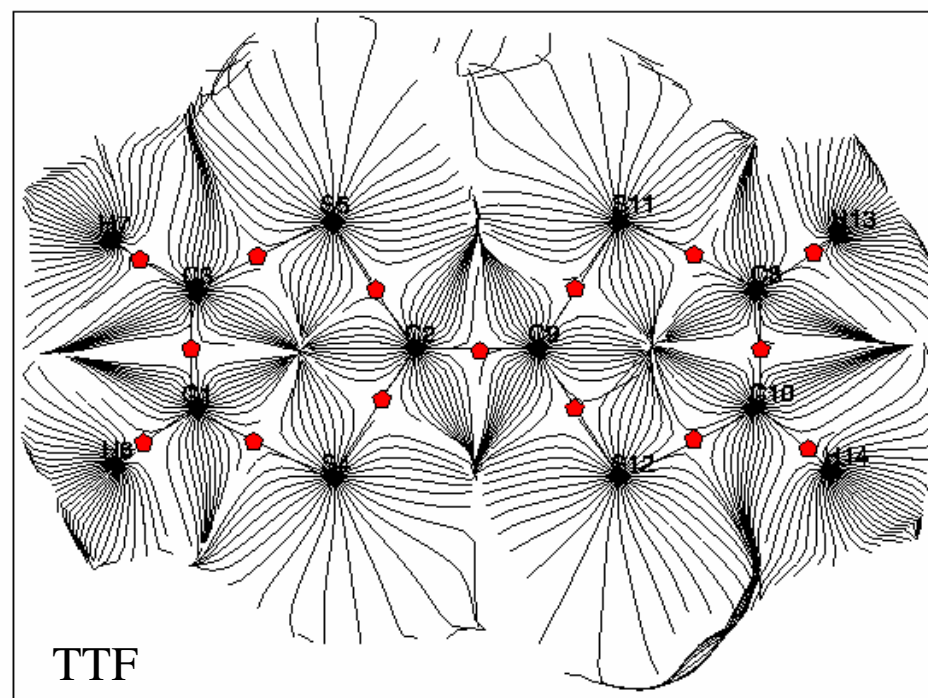
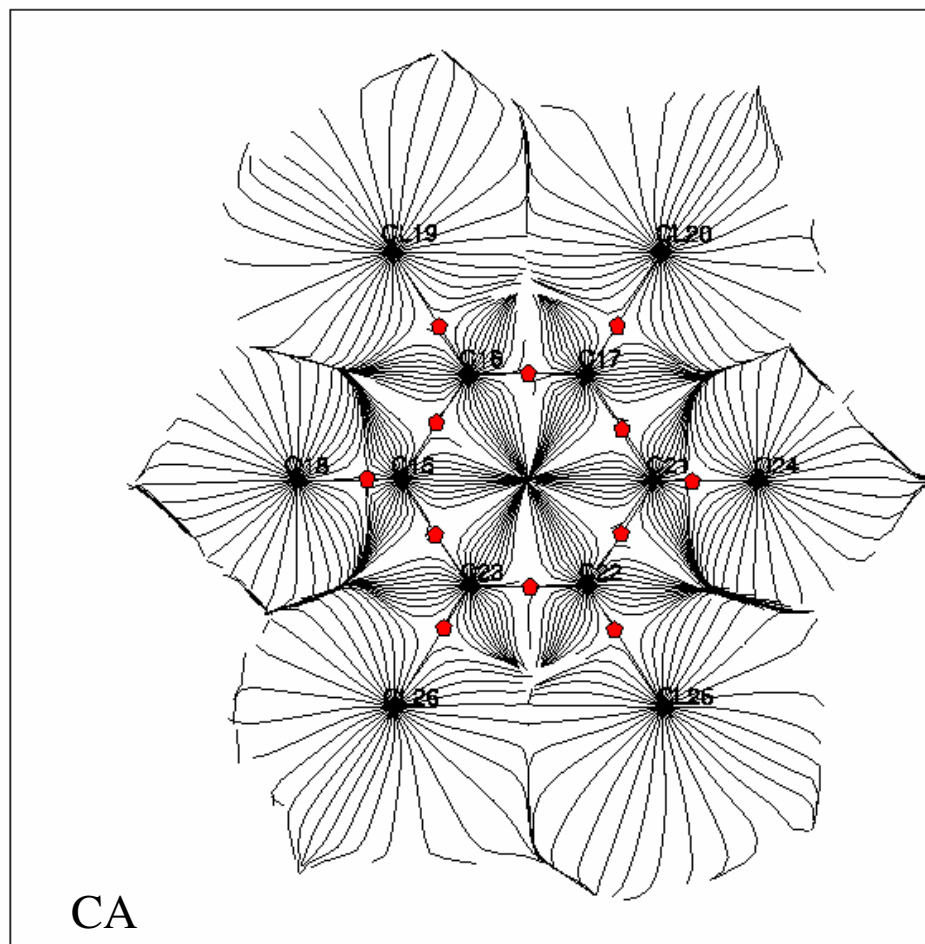


# Topological Charges and Volumes

## NEWPROP

Souhassou et al. J. Appl. Cryst., 32, 210 (1999)

Charge : 
$$q_e = \int_{\Omega} \rho(\vec{r}) d\vec{r}$$

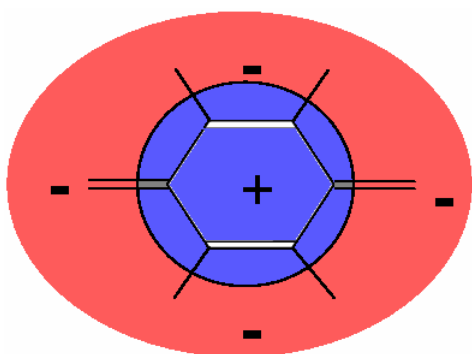
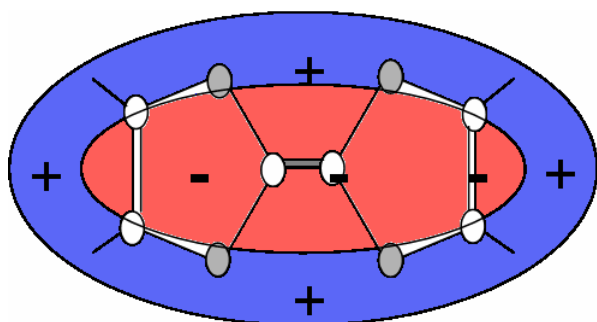


**Topological Charge neutral 0.21 ionic 0.74**



# Charge representation of the TTF and CA ions

$$q_{nette} = Q_{neutre} - \int_{\Omega_{atomique}} \rho(\vec{r}) \cdot d\Omega$$



Atom	105K	15K
C1/C8	-0,26	-0,53 / -0,54
C2/C9	-0,36	-0,23 / -0,24
C3/C10	-0,38	-0,29 / -0,29
S4/S11	0,26	0,38 / 0,38
S5/C12	0,37	0,50 / 0,50
H6/C13	0,20	0,31 / 0,31
H7/H14	0,27	0,26 / 0,26
C15/C21	0,71	0,92 / 0,90
C16/C22	-0,02	0,05 / 0,04
C17/C23	-0,01	0,30 / 0,29
O18/O24	-0,74	-1,09 / -1,07
Cl19/Cl25	-0,07	-0,19 / -0,19
Cl20/Cl26	0,02	-0,34 / -0,33

## Comparaison between all methods

	RT	90K	40K	15K	Delta Q
<b>Topological Charges</b>		<b>0.21</b>		<b>0.74</b>	<b><u>0.53</u></b>
<b>Pv-kappa</b>		<b>0.14</b>		<b>0.67</b>	<b><u>0.53</u></b>
<b>Multipolar</b>		<b>0.06</b>		<b>0.65</b>	<b><u>0.59</u></b>
DFT	0.48	0.54	0.64	0.63	0.09
VASP LDA		0.54		0.67	0.13
VASP PBE		0.58		0.64	0.06
<b>VASP New hybrid functional</b>		<b>0.10</b>		<b>0.80</b>	<b>0.70</b>

Garcia , Dahaoui et Al Faraday discussions 135 , 2007

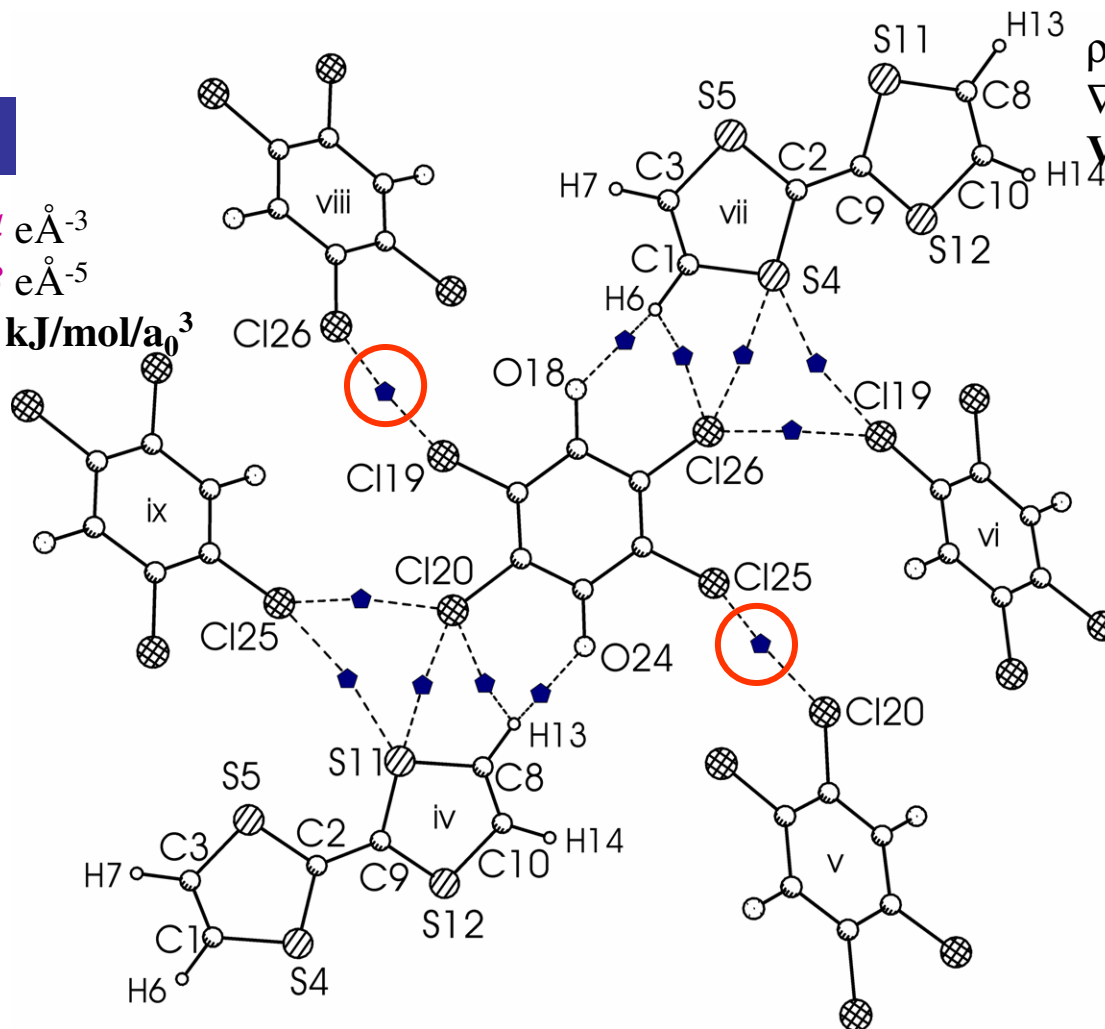
# Nature of the Cl...Cl Interactions ?

**15 K**

$$\rho = 0,05 / 0,04 \text{ e}\text{\AA}^{-3}$$

$$\nabla^2\rho = 0,51 / 0,53 \text{ e}\text{\AA}^{-5}$$

$$V_{PC} = -8,8 / -8,3 \text{ kJ/mol/a}_0^3$$



**105 K**

$$\rho = 0,05 / 0,04 \text{ e}\text{\AA}^{-3}$$

$$\nabla^2\rho = 0,50 / 0,56 \text{ e}\text{\AA}^{-5}$$

$$V_{PC} = -8,8 / -8,4 \text{ kJ/mol/a}_0^3$$

**15 K**

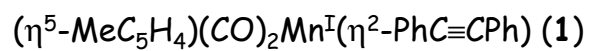
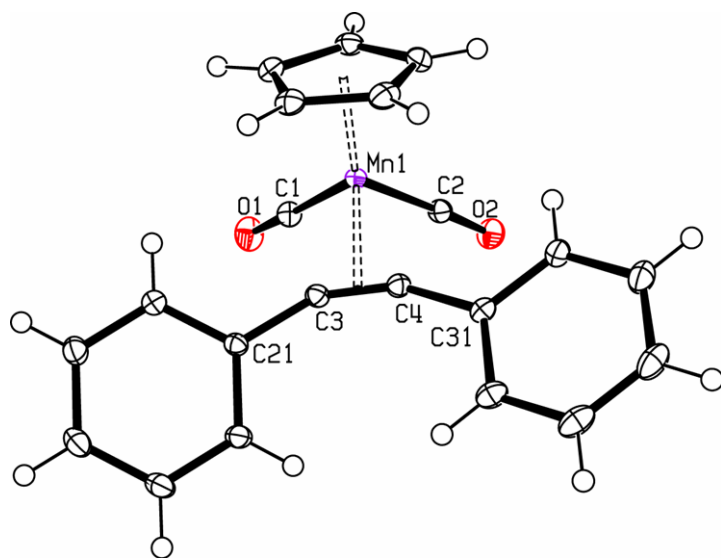
$$\rho = 0,04 / 0,04 \text{ e}\text{\AA}^{-3}$$

$$\nabla^2\rho = 0,56 / 0,55 \text{ e}\text{\AA}^{-5}$$

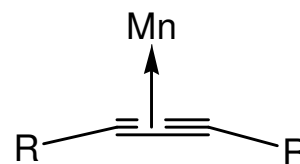
$$V_{PC} = -8,0 / -8,7 \text{ kJ/mol/a}_0^3$$

iv)-1+x, -1+y, z; v)1/2+x, 1/2-y, 1/2+z; vi)1/2+x, 3/2-y, 1/2+z; vii)x, 1+y, z; viii)-1/2+x, 3/2-y, -1/2+z; ix)-1/2+x, 1/2-y, 1/2+z

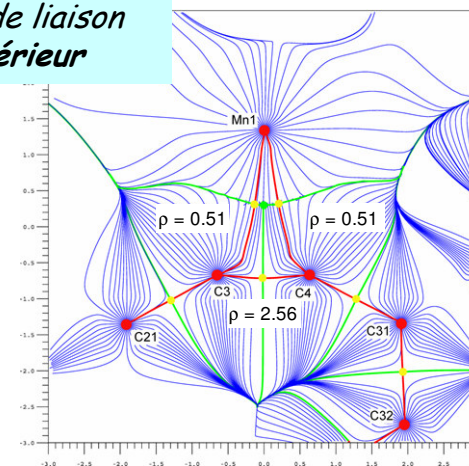
# Applications a la liaison metal ligand ( voir Poster N Luga )

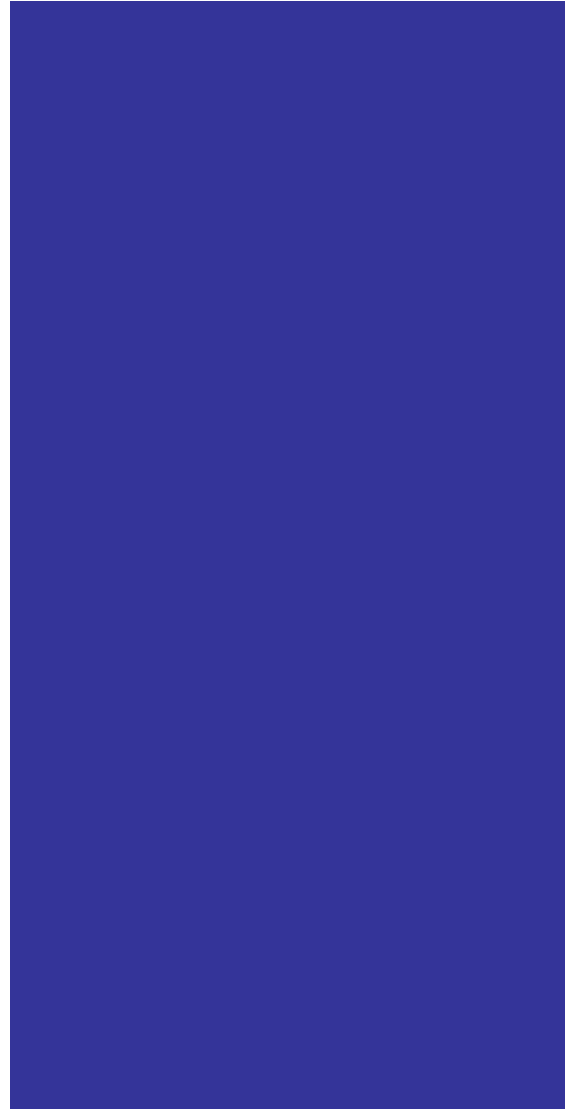


Archétype d'un complexe  $\eta^2$ -alcyne où le métal de transition est dans un bas degré d'oxidation, et où l'alcyne est formellement **donneur à 2 électrons**



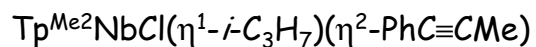
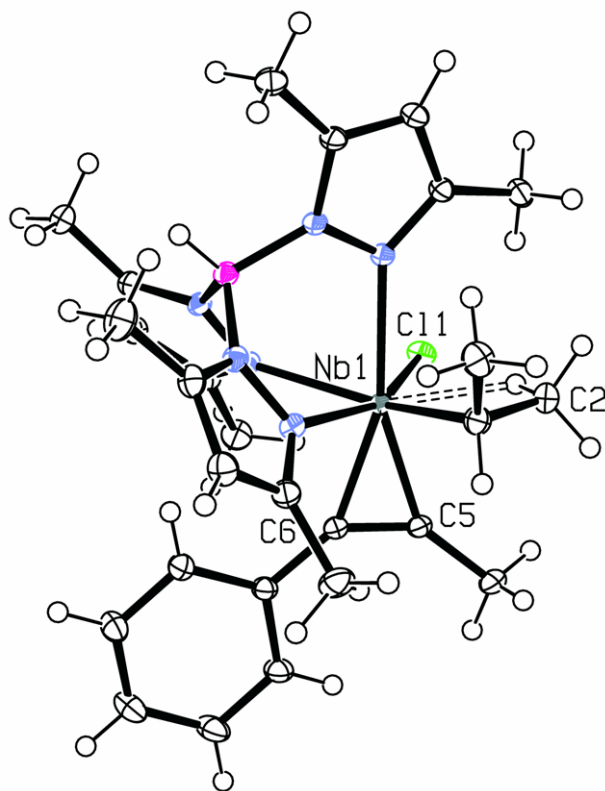
*Courbure des chemins de liaison vers l'intérieur*





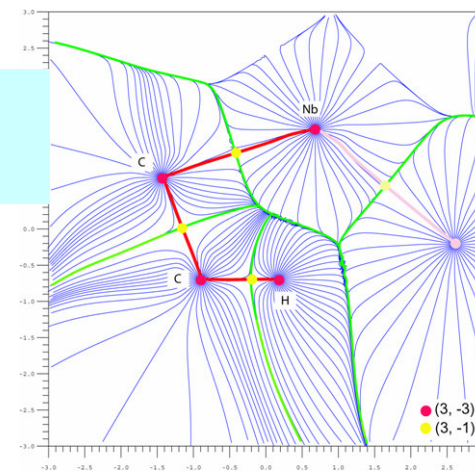
# Applications a la liaison metal ligand ( voir Poster N Lugan )

Complexe du Nb (Z = 41)  
Présence d'une liaison agostique C-H

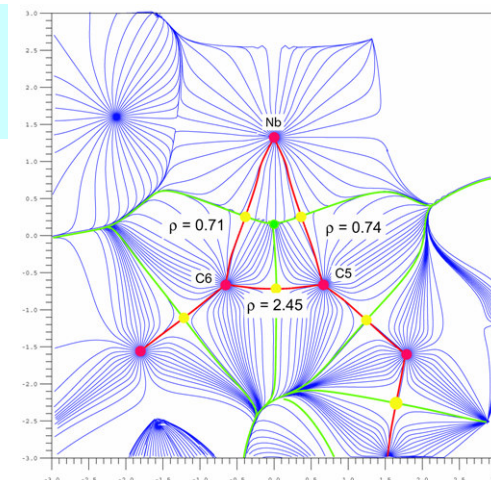
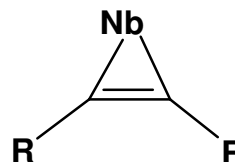


Complexe  $\eta^2$ -alcyne où le métal de transition est dans un haut degré d'oxidation, et où l'alcyne est formellement donneur à 4 électrons

Aucun point critique de liaison n'est mis en évidence pour l'interaction agostique C-H

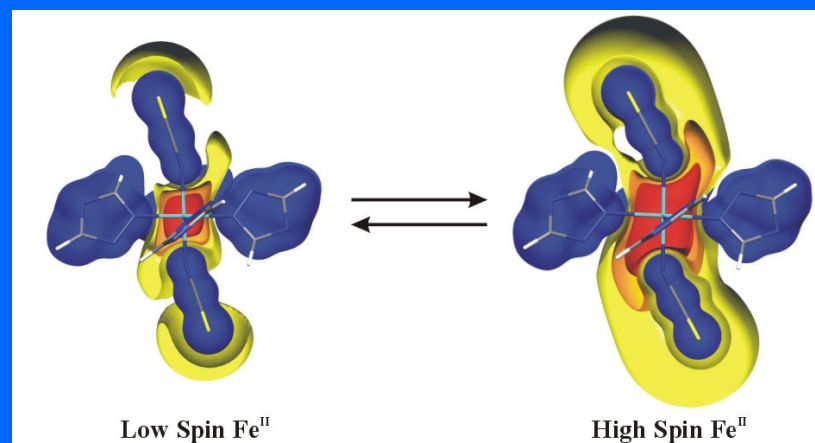
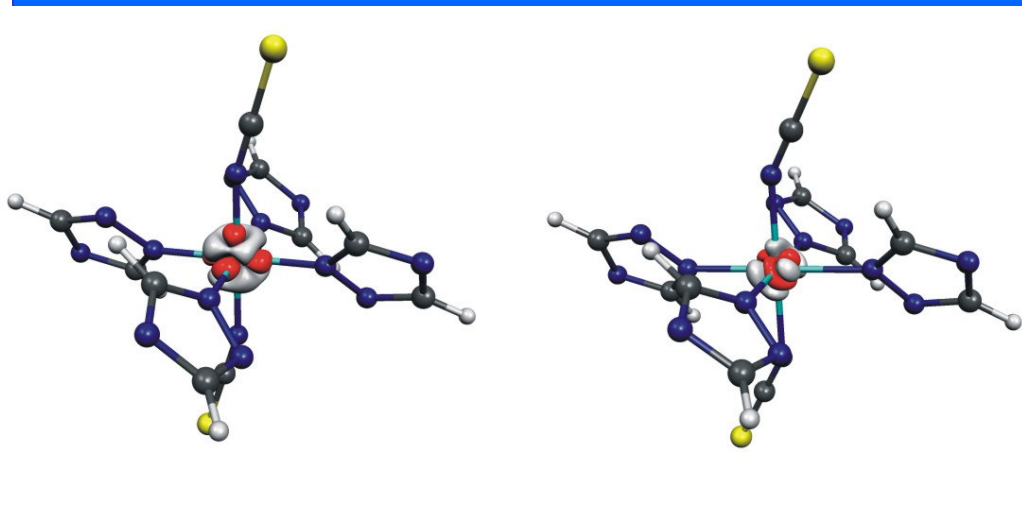


Courbure des chemins de liaison vers l'extérieur



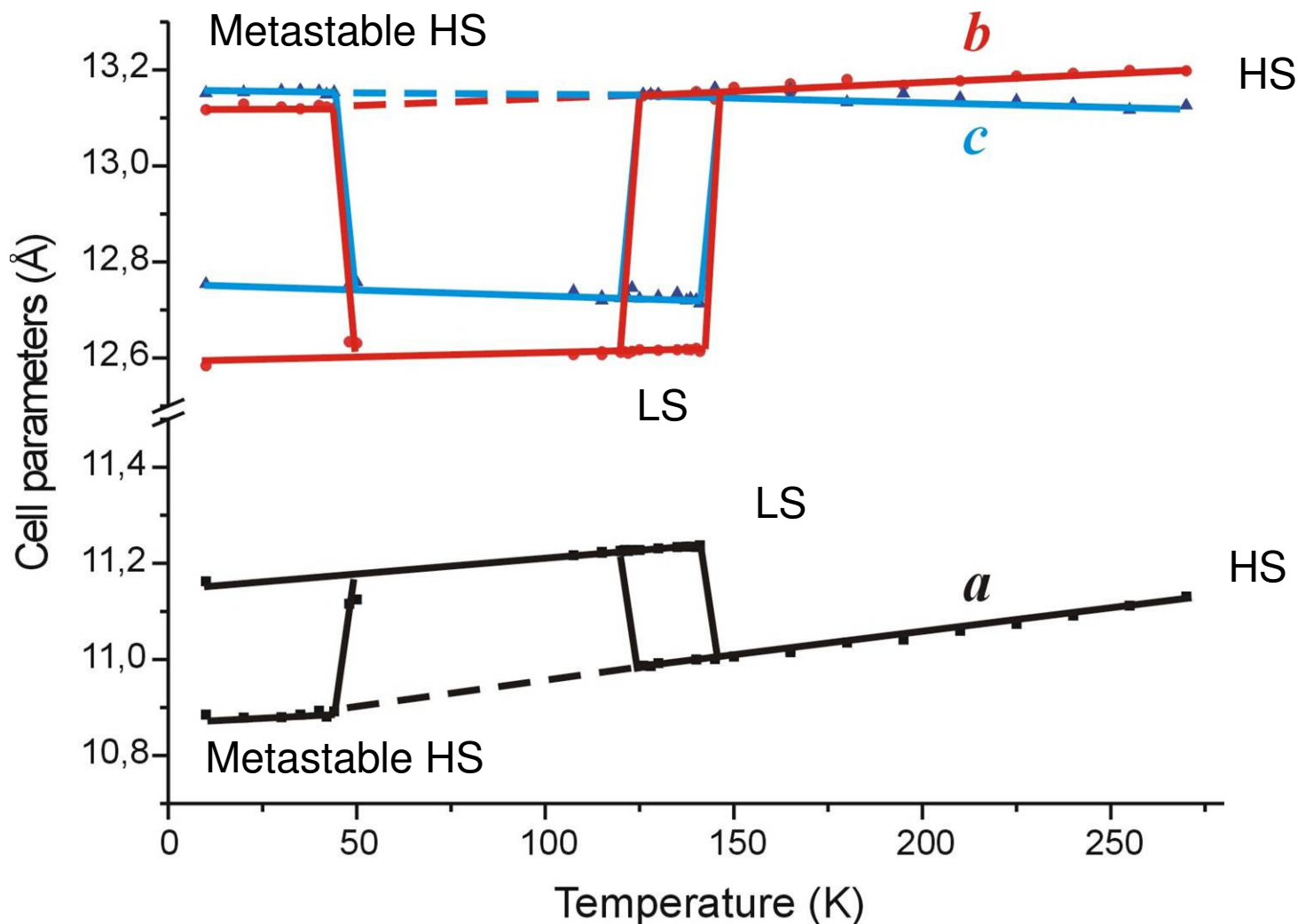
La topologie de la DDE pour l'alcyne coordonné au métal est celles que l'on peut attendre d'un métala-cyclopropène.

# Thermally and light induced spin transitions of $\text{Fe}(\text{btr})_2(\text{NCS})_2 \cdot \text{H}_2\text{O}$ : first examples of charge density of metastable states





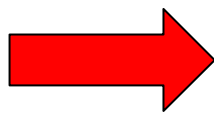
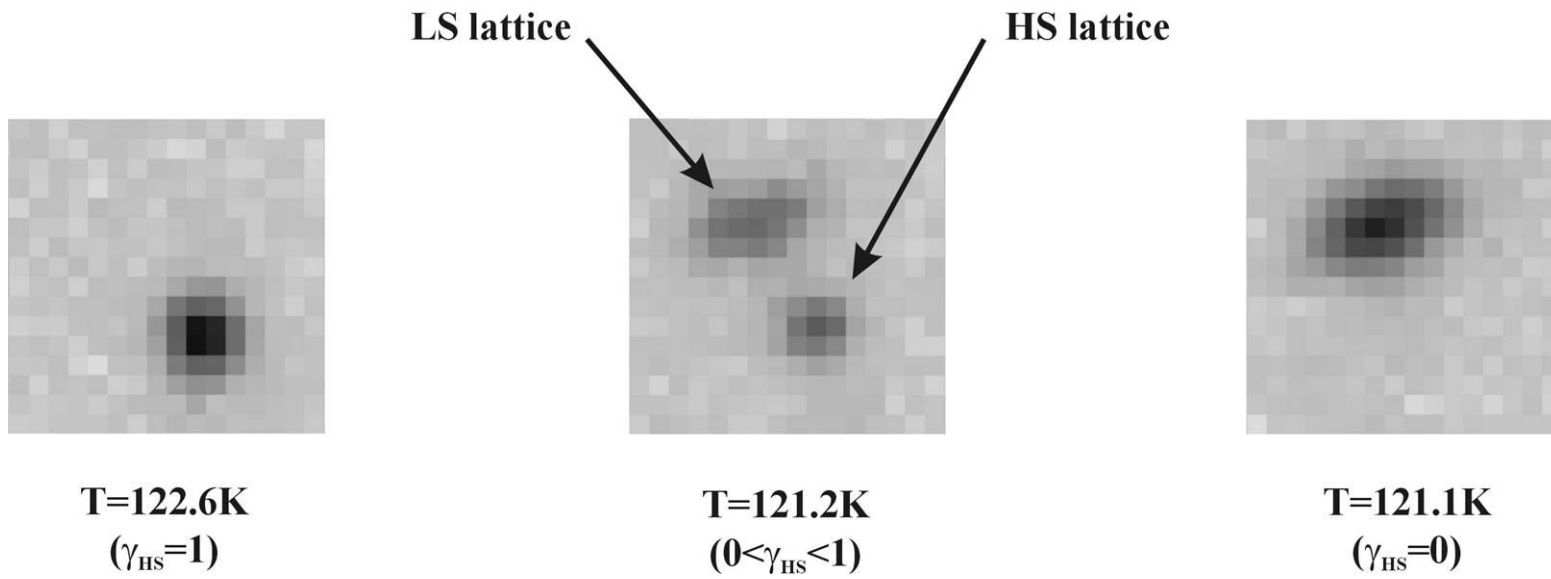
# $\text{Fe}(\text{btr})_2(\text{NCS})_2$ Cell parameters variation during the thermal and photo induced transitions



S. Pillet et al, *Eur. Phys. J. B* 38, 2004,541  
,*Phys. Rev. B* 74, 2006, 140101

# Thermal transition

Varying Temperature and Following a reflection when the HS  $\leftrightarrow$  S thermal transition occurs

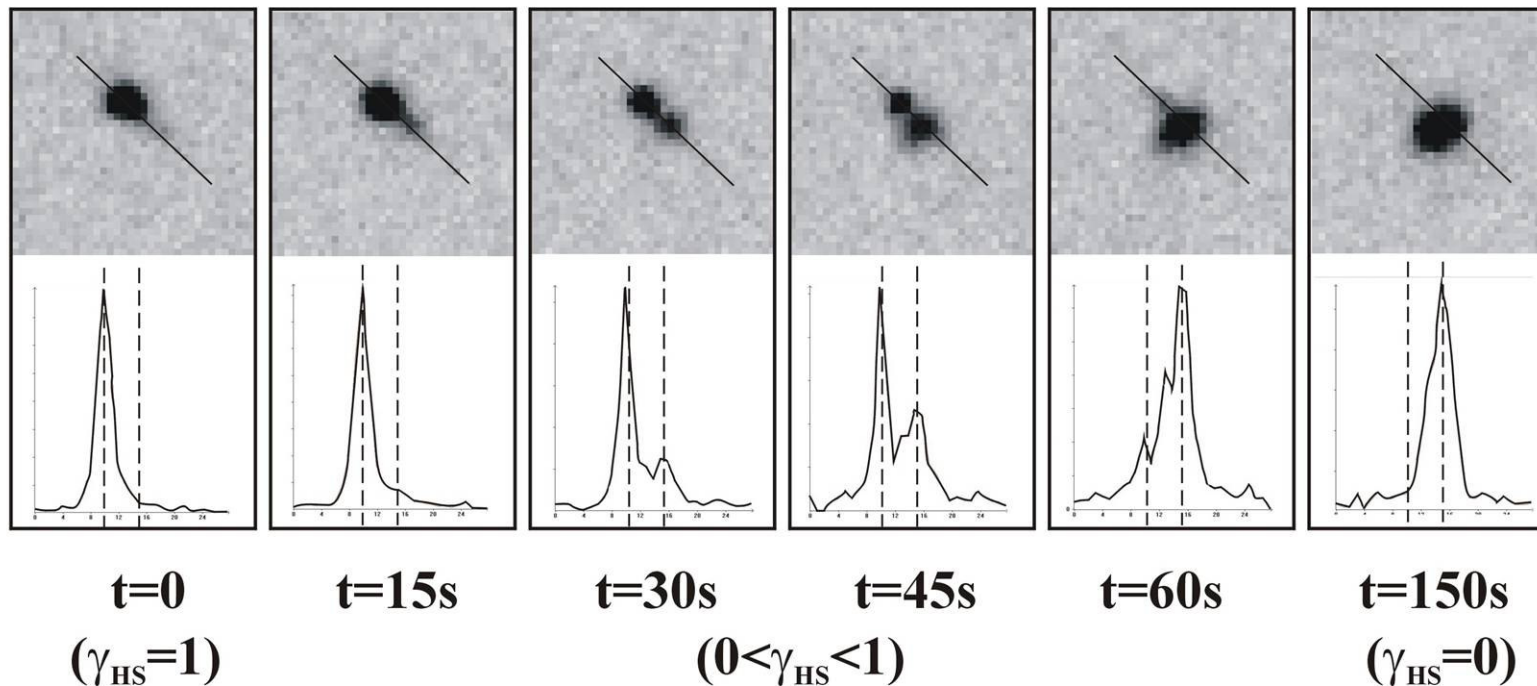


Long range order is kept during the transition

Spin like domain formation

# Dynamic of the thermal transition

Following the transition versus time at  $T_c = 117.2K$

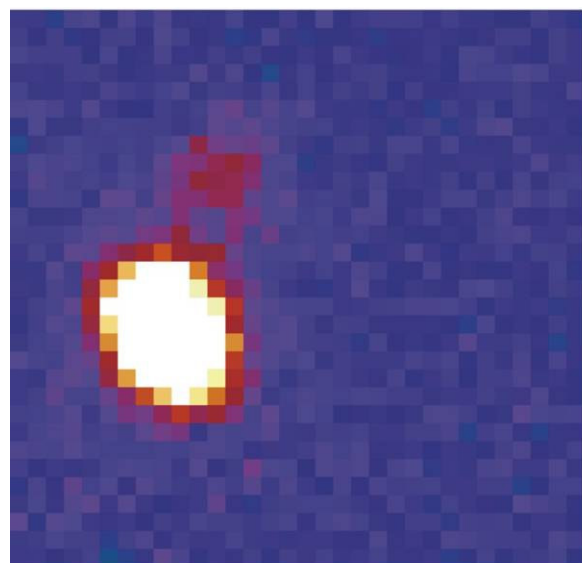


Long range Order and coexistence of LS and HS domains

S. Pillet, J Hubsch and Lecomte, *Eur. Phys. J. B*, 2004, 38, 541.

**Fe(btr)<sub>2</sub>(NCS)<sub>2</sub> :**

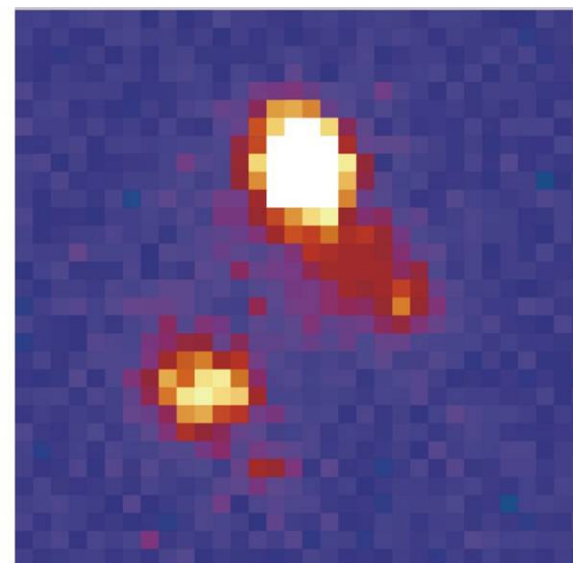
**Excitation at 10 K**



**100% LS**

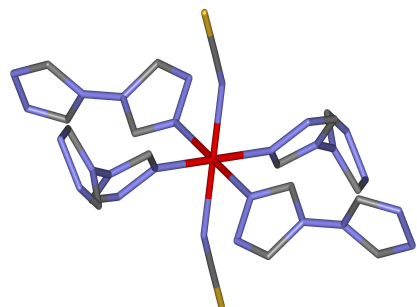
During excitation

$\lambda = 488 \text{ nm}$



**~80% HS**

# Lattice Dynamics during the LIESST spin conversion



Laser conditions

$T = 15 \text{ K}$

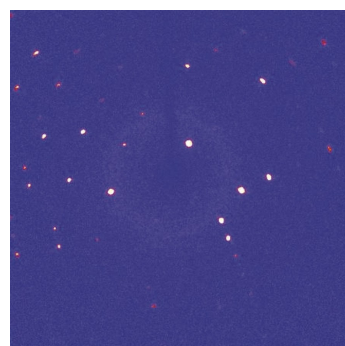
$\lambda = 488 \text{ nm}$

$P = 92 \text{ mW}$

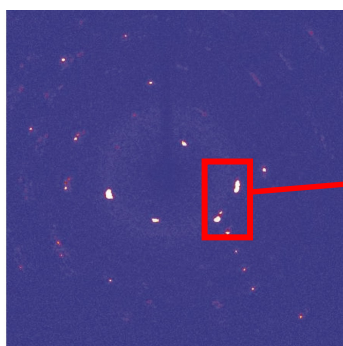
Data collection at home  
Oxford Diffraction XCalibur

$\Delta\omega = 5^\circ$

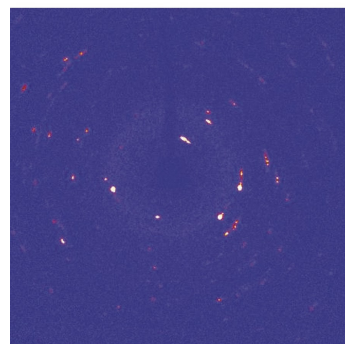
$\Delta t = 5 \text{ s}$



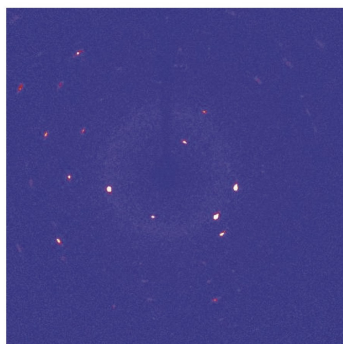
BS :  $t = 0 \text{ min}$



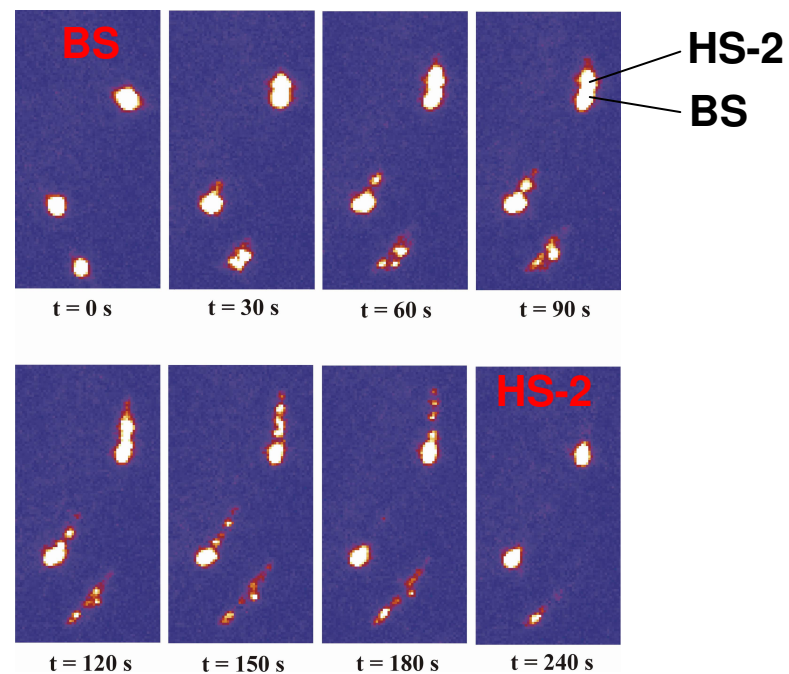
$t = 2 \text{ min}$



$t = 5 \text{ min}$



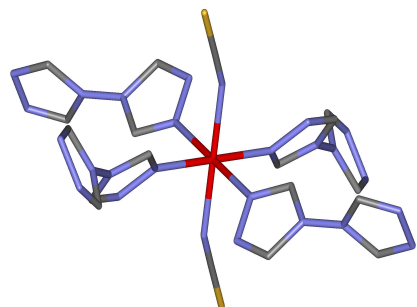
$t = 7 \text{ min}$



Long range Order and coexistence of  
LS and HS domains



# Lattice Dynamics during the LIESST spin conversion



Laser conditions

$T = 15 \text{ K}$

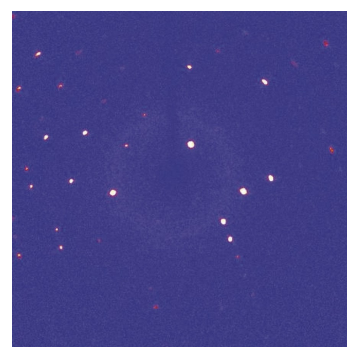
$\lambda = 488 \text{ nm}$

$P = 92 \text{ mW}$

Data Collection

$\Delta\omega = 5^\circ$

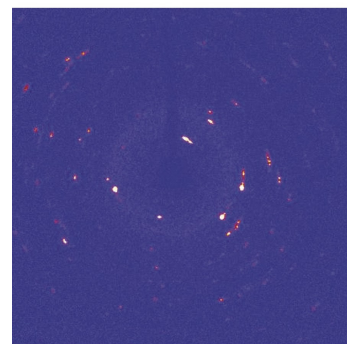
$\Delta t = 5 \text{ s}$



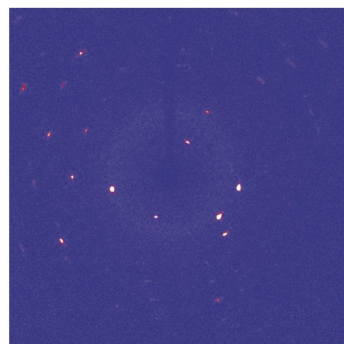
BS : t = 0 min



t = 2 min

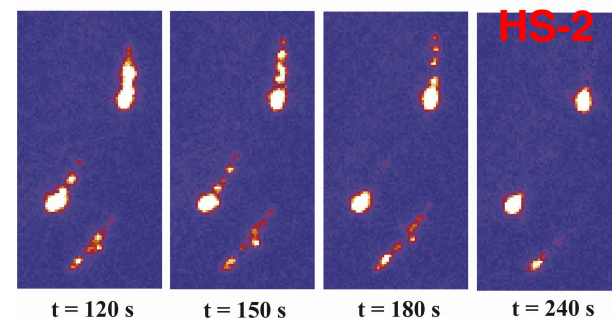
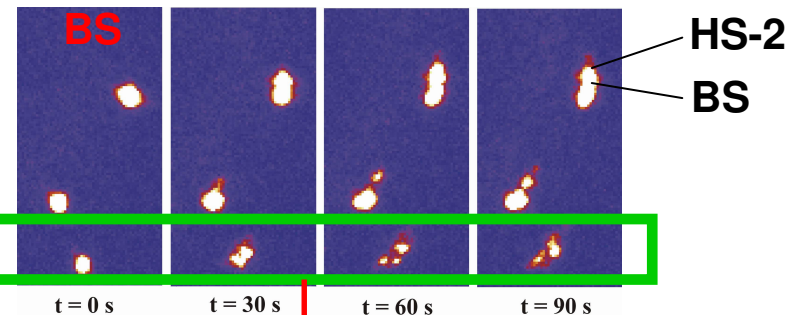


t = 5 min



t = 7 min

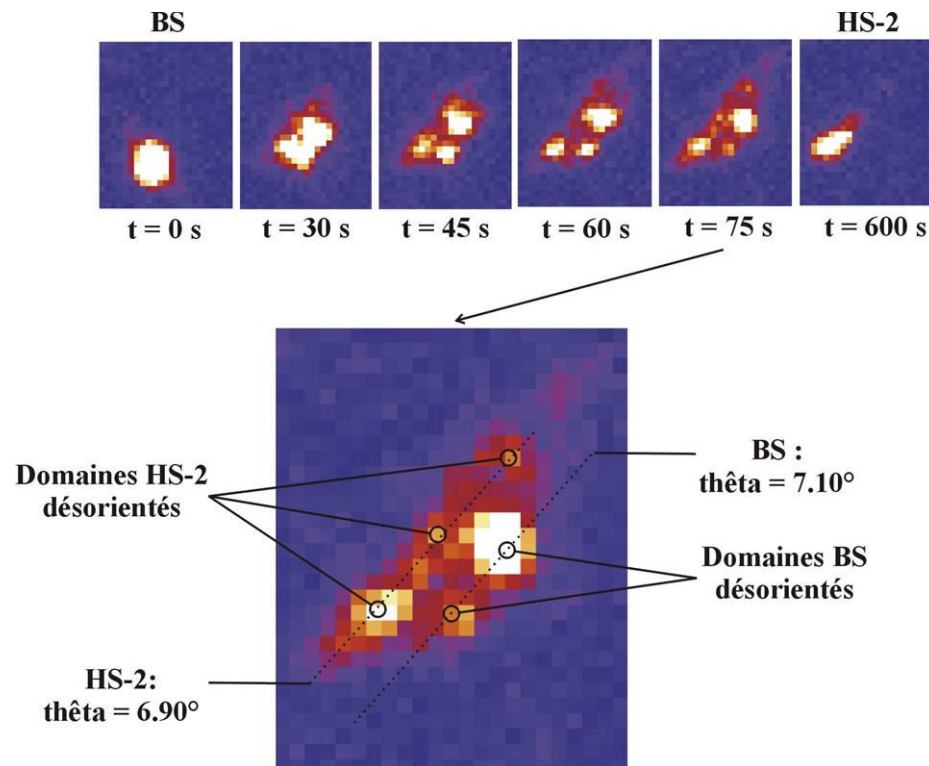
(0 2 -4)  
reflection



Long range Order and coexistence of LS and HS domains

# Lattice Dynamics during the LIESSTspin conversion

## ◆ Evolution of the (0 2 -4) Bragg reflection



HS domains grow in slightly different orientations; the LS matrix reacts by creating desoriented domains

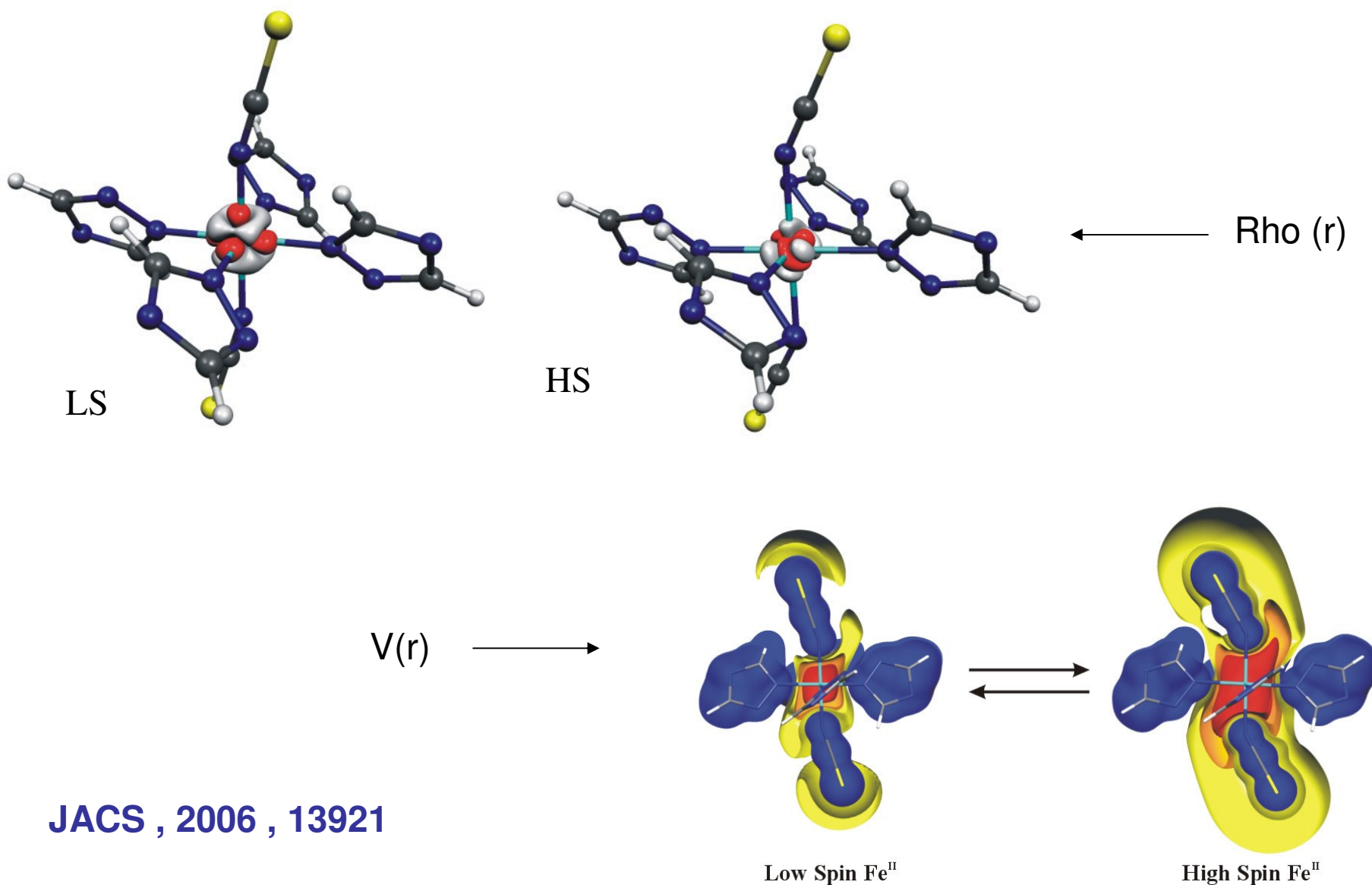
When the transition is complete ( $t = 600$ s) the HS domains merge to reconstruct one unique HS lattice

# Experimental Charge density of the 15K Metastable HS and LS Fe<sup>2+</sup> states

	LS	HS
Space group	C2/c	C2/c
V (Å <sup>3</sup> )	1790.2(1)	1881.7(2)
Crystal size mm	0.32*0.20*.013 0.36*0.18*0.12	
Measured reflections	26316	16975
Unique reflections	6407	3879
R <sub>int</sub> (I)	0.034	0.033
Sinθ/λ <sub>max</sub> (Å <sup>-1</sup> )	0.98	0.85
	<i>IAM refinement</i>	
R (all data)	0.039	0.035
	<i>Multipolar refinement</i>	
R (S < 0.7Å <sup>-1</sup> / all data)	0.018 / 0.032	0.022 / 0.032



# 3D representation of the 3d electron density in the vicinity of the iron atom



JACS , 2006 , 13921

# Multipole model allows d ORBITAL POPULATIONS calculation

$$\rho_d = \sum_{i=1}^5 P_i d_i^2 + \sum_{i=1}^5 \sum_{j>i}^5 P_{ij} d_i d_j = \sum_{l=0}^{l_{\max}} \kappa'^3 R_l(\kappa' r) \sum_{m=0}^{+l} \sum_p P_{lmp} y_{lmp}(\theta, \varphi)$$

$$d_i = R(r) Y_{lm\pm}$$

MATRIX RELATING  
P<sub>lm</sub> TO P<sub>i</sub>

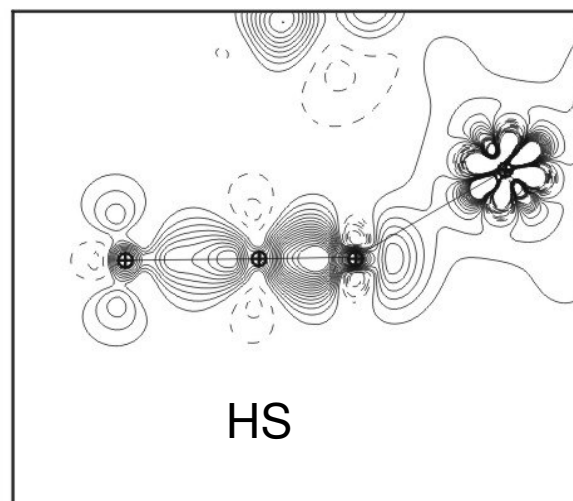
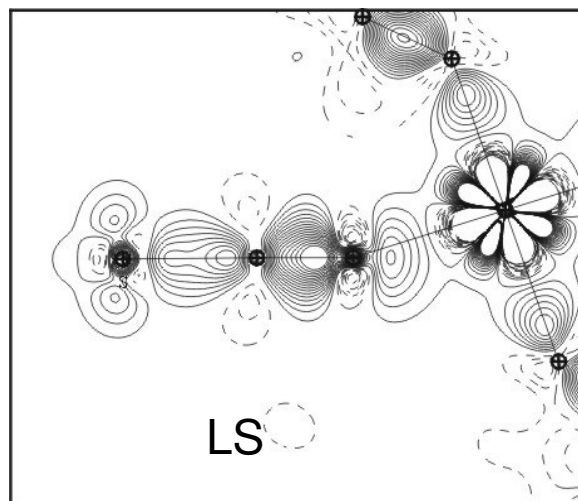
$$P_i = M^{-1} P_{lm\pm}$$

d orbital population

MULTIPOLE  
POPULATIONS

# 3d atomic orbital populations of iron in LS and HS states. Crystal field hypothesis : pure octahedral symmetry.

	$dx^2-y^2$	$dz^2$	$d_{xy}$	$dxz$	$dyz$
Total 3d					
LS 6.26	0.40	0.20	1.50	2.22	1.94
LS crystal field 6	0	0	2	2	2
HS 6.14	0.95	1.49	1.59	0.94	1.17
HS crystal field 6	1	1	1.33	1.33	1.33



# Charge density and electrostatic properties of a zeolithe like material (these E Aubert , JPCS,2004,65,1943 )

**AlPO<sub>4</sub>-15**

**(NH<sub>4</sub>Al<sub>2</sub>(OH)(H<sub>2</sub>O)(PO<sub>4</sub>)<sub>2</sub>·H<sub>2</sub>O )**

**very narrow voids :**       $\text{Ø} = 4,4 \text{ \AA} \times 5,3 \text{ \AA} // [001]$   
    $\text{Ø} = 3,5 \text{ \AA} \times 4,7 \text{ \AA} // [010]$

**Then very low adsorption capability ( H<sub>2</sub>O and O<sub>2</sub> only)**

**Molecules or ions trapped in the framework during the synthesis:**

**1 free water molecule:**

**w11**

**1 water bound to Al :**

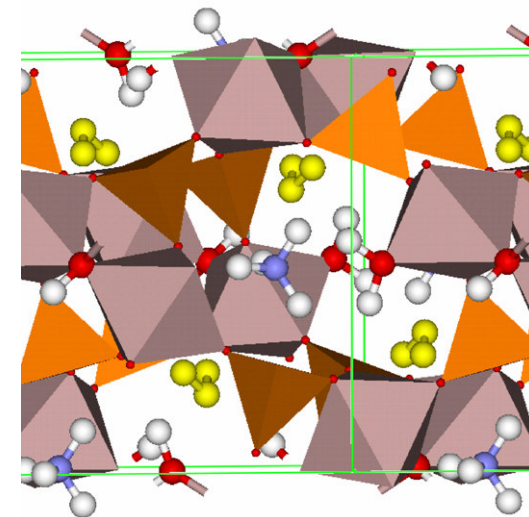
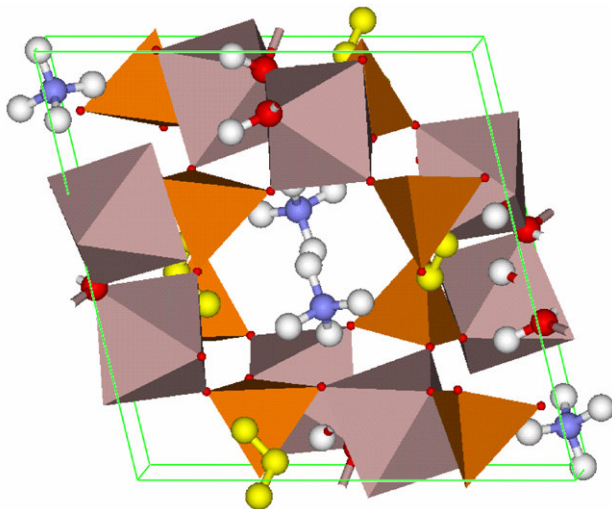
**w10**

**1 ammonium cation :**

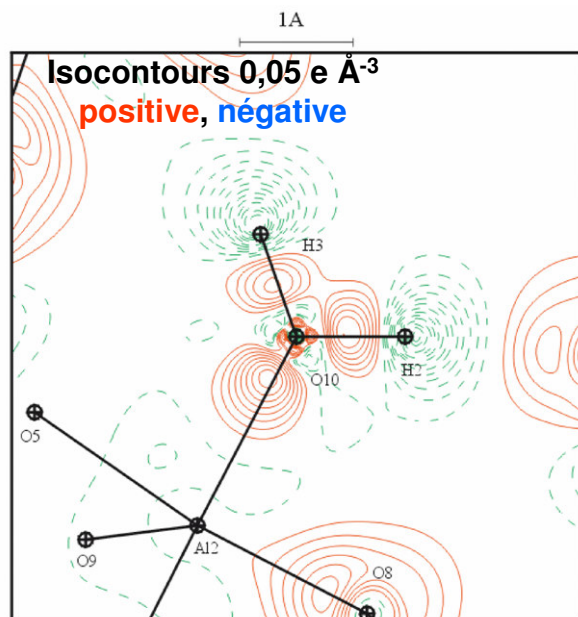
**NH<sub>4</sub><sup>+</sup>**

**1 OH anion bounded to 3 Al**

**OH<sup>-</sup>**

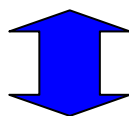


# Water – Framework INTERACTIONS

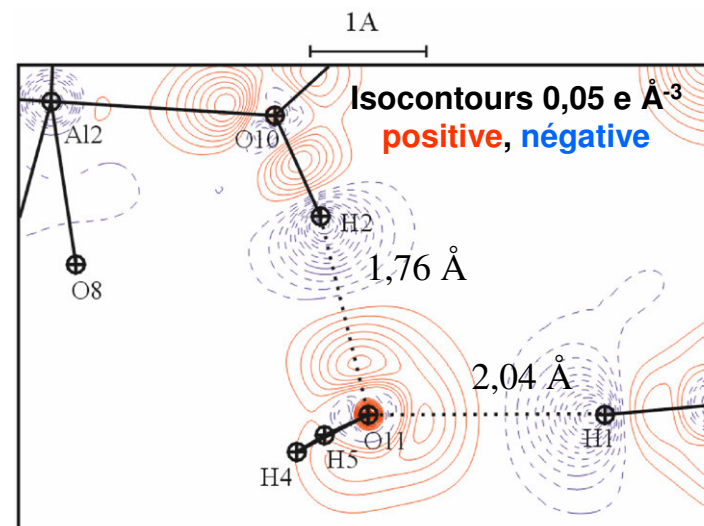


hydrogen bonds

Deformation



Environment



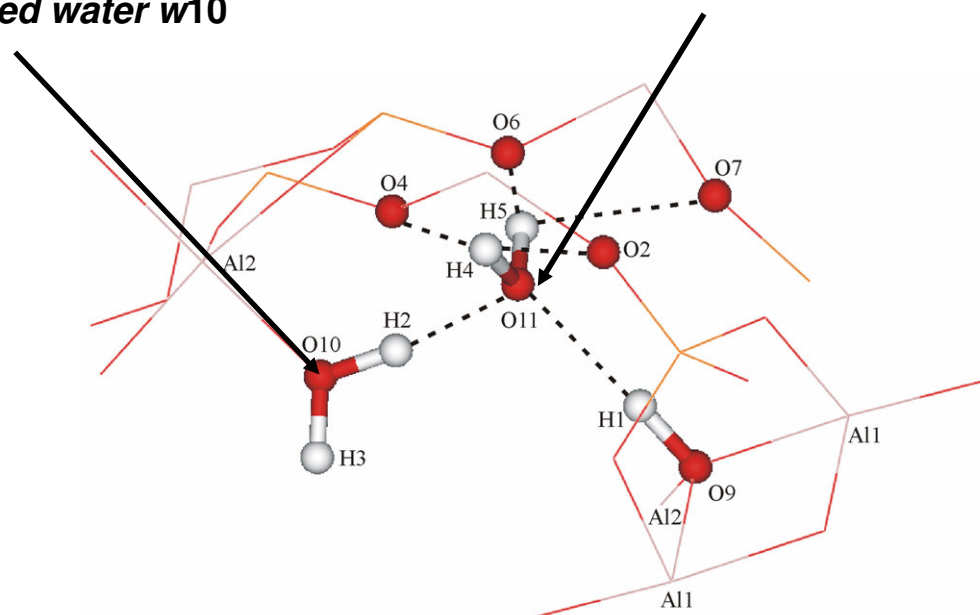
*Al Bounded water w10*

*'free' water W11*

**Topological Analysis : molecular Dipole moments estimation**

	w10	w11
Charge (e)	-0,07	0,14
Dipôle total (D)	2,76	2,06

**Comparaison : Gaz = 1,85 D**



# H2O –framework electrostatic interaction energy

$$E_{elec} = \int \rho_{H2O}(\vec{r}) V_{Host}(\vec{r}) d\vec{r}$$

$$\rho(\vec{r}')$$

**CHARGES  
DISTRIBUTION**  
(nuclei, e<sup>-</sup>)

$$V(\vec{r}) = \int \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} d\vec{r}'$$

**ELECTROSTATIC  
POTENTIAL**

**2 models for  $\rho(\vec{r}')$**

**CONTINUED CHARGE DISTRIBUTION**  
*Multipolar model*

$$\rho(\vec{r}') = \sum_j Z_j \delta(\vec{r}' - \vec{R}'_j) - \rho_{elec}(\vec{r}')$$

**DISCRETE CHARGE DISTRIBUTION**

$$\rho(\vec{r}') = \sum_j Q_j \delta(\vec{r}' - \vec{R}'_j)$$

$Q_j$  {  
 Formal Charges Al<sup>3+</sup>, Si<sup>4+</sup>, O<sup>2-</sup>, ...  
 Kappa Charges  
 Topological Charges



# Electrostatic potential at the guest sites

## Bertaud, Stewart Method

:combination of direct and reciprocal lattices calculations

$$V_{Tot}(\vec{r}) = V^{IAM}(\vec{r}) + \Delta V(\vec{r}) - V_0$$

$$\int \frac{\rho^{IAM}(\vec{r}')}{|\vec{r} - \vec{r}'|} d\vec{r}'$$

Direct Lattice : local calculation

$$\frac{-1}{\pi V_{maille}} \sum_{\vec{H} \neq 0}^{H_{max}} \frac{F_{multi}(\vec{H}) - F_{IAM}(\vec{H})}{H^2} e^{-2\pi i \vec{H} \cdot \vec{r}}$$

Reciprocal Lattice : périodic calculation

$$-\frac{2\pi}{3V_{maille}} \int r^2 \rho_t(r) d\vec{r}$$

Mean potential

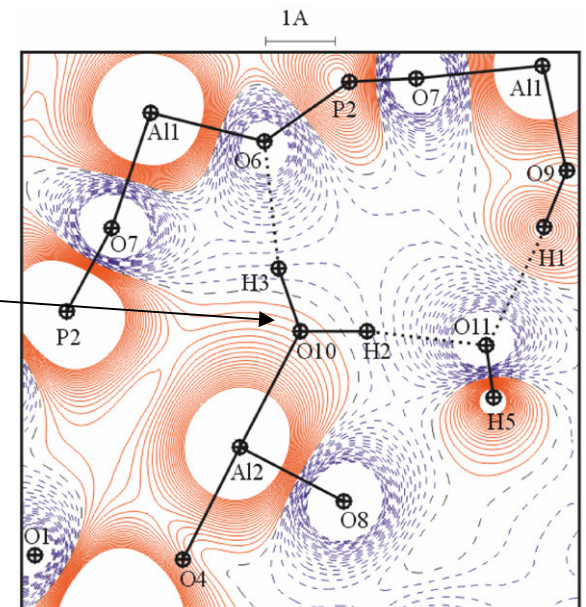
$V_{Host}(\vec{r})$  calculation of potential created by framework at H<sub>2</sub>O site

$$V_{HOST}(\vec{r}) = V_{tot}(\vec{r}) - V_{H_2O}(\vec{r})$$

With

$$V_{H_2O} = \int \frac{\rho^A(\vec{r}')}{|\vec{r} - \vec{r}'|} d\vec{r}'$$

calculated either in direct or reciprocal space



# Interaction energy calculation of guest molecules

## 1) POINT CHARGE MODEL, very simple model

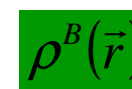
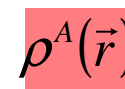


$$E^{A/B} = \sum_p Q_p^A \cdot V_p^B(Q_q^B)$$

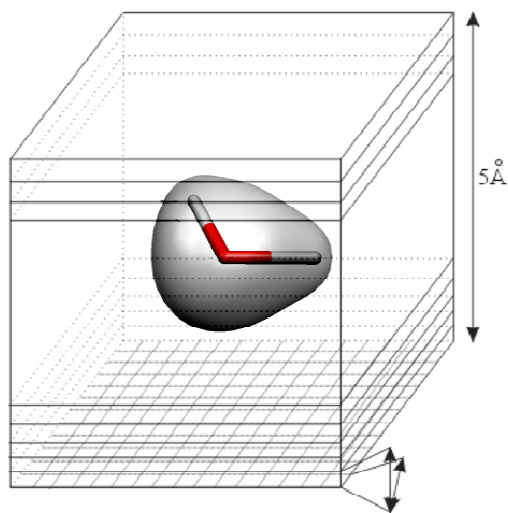


: formal or Kappa or topological charges

## 2) MULTIPOLAR MODEL ,elaborated model



$$E^{A/B} = \int_V \rho^A(\vec{r}) V^B(\vec{r}) d\vec{r}$$



**Numerical evaluation on a grid**

$$E = \sum_{i,j,k} \rho_{ijk}^A V_{ijk}^B$$



## Water Host Electrostatic Energy Interaction

---

	OH anion	NH <sub>4</sub> <sup>+</sup>	W (Al)	W11 (guest)
E(Q Topo.)	-0,84	-0,40	-0,29	-0,20
<b>E(<math>\rho(r)</math>)</b>	<b>-0,97</b>	<b>-1,03</b>	<b>-0,57</b>	<b>-0,46</b>

---

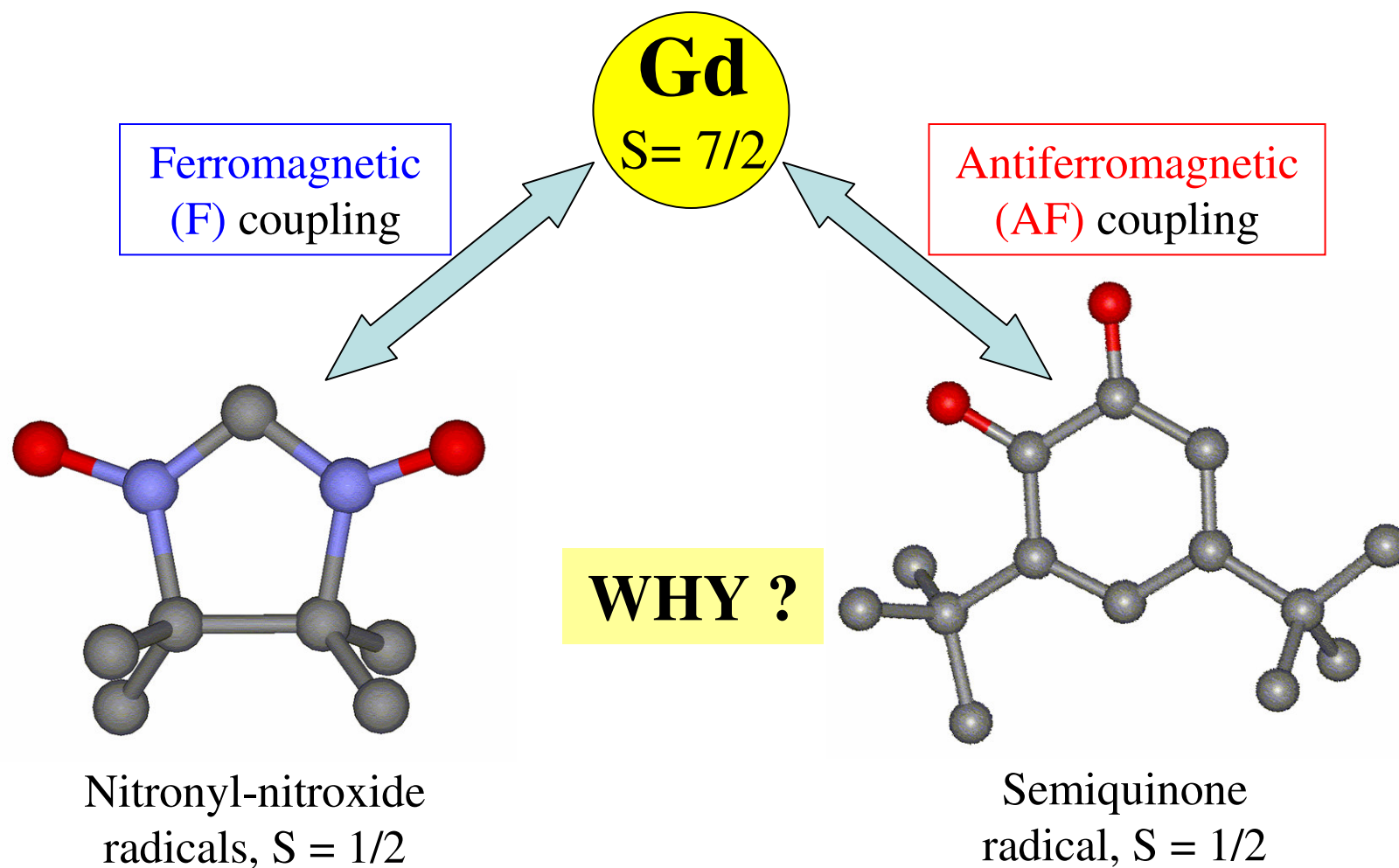
**BUT** relaxation (electric, geometric) of the framework  
not included in the calculation

# OUTLINE

- **Structure factor and electron density analysis**  
**Multipole Model , thermal motion analysis and electrostatics**
- **Applications to**
  - Quantum Chemistry :Bonding**
  - Phase transitions :TTF CA**
  - Metastable states :Thermal and photoexcited molecules**
  - Material Science : Electrostatics in zeolite**
- **Problems with heavy elements and need of SR**

# Necessity of Synchrotron radiation: example of rare earths

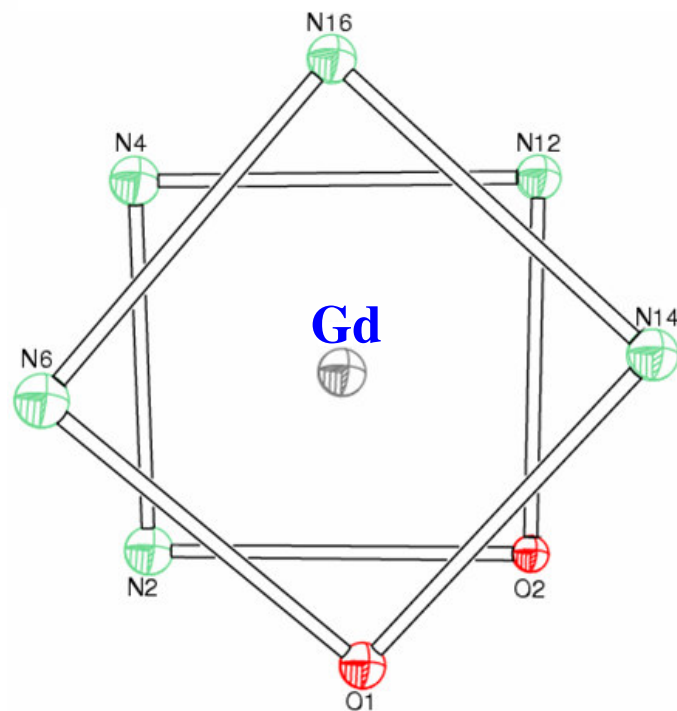
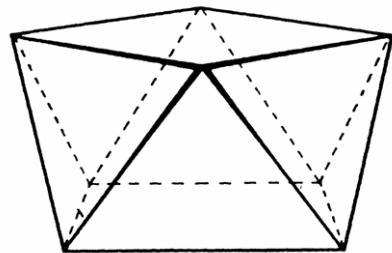
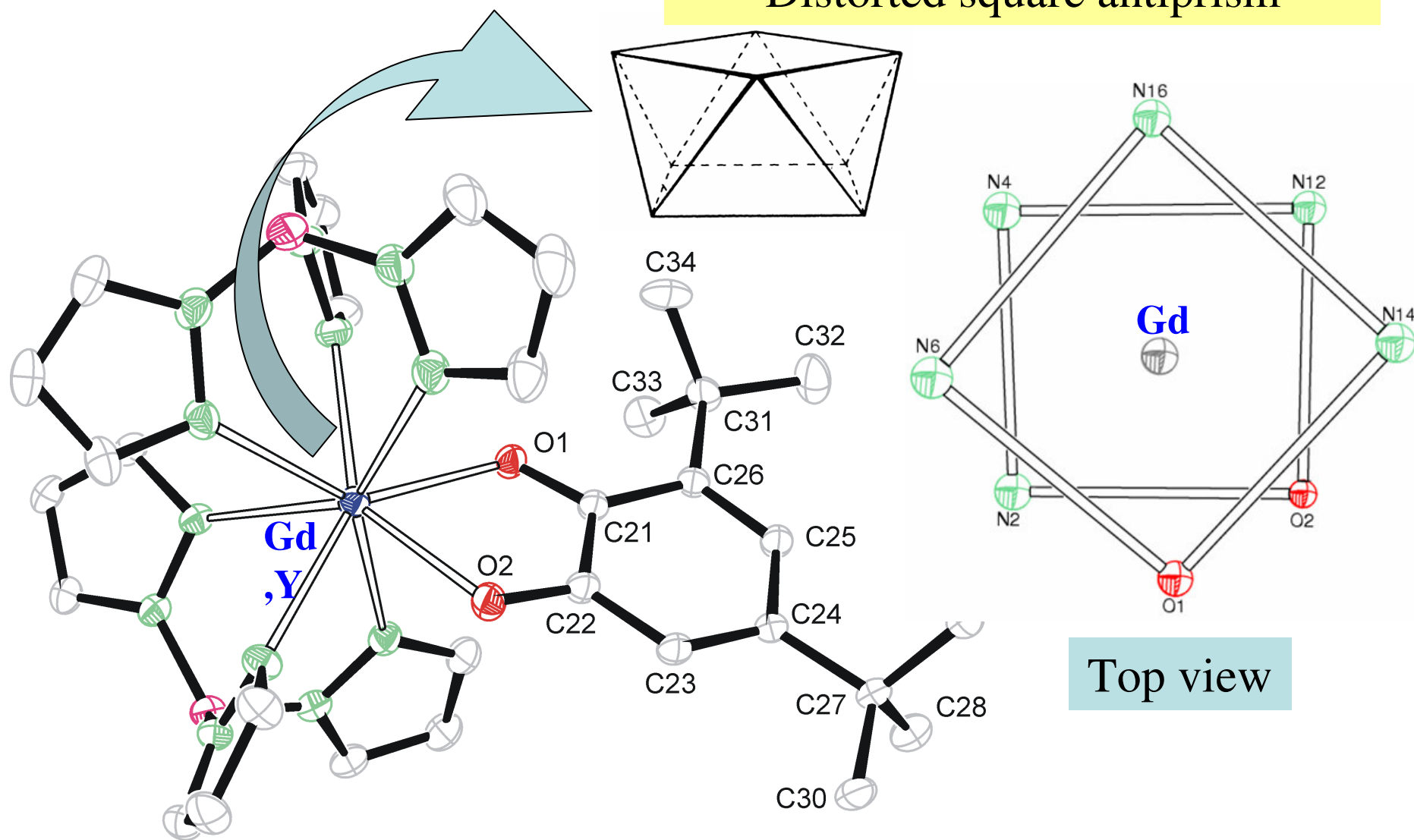
## These N Claiser



# Charge density of Y or Gd complexes

These N Claiser and JPCS 2004 , 65 , 1927

Distorted square antiprism



Top view

# Data collection: Y complex

## X-ray data collection (Nonius KappaCCD)

Chemical formula	YO <sub>2</sub> N <sub>12</sub> C <sub>32</sub> B <sub>2</sub> H <sub>40</sub>
Space group, cell setting	P2 <sub>1</sub> /c, monoclinic
Temperature (K) , Wavelength (Å)	106, 0.71074 Mo(Kα)
a, b, c (Å) ; β (°)	12.5943(1), 14.8920(1), 19.5690(1) ; 98.428(1)
Scan method, Oscillation width (°)	φ and ω rotations, 2.0
Exposure time per degree (s)	150 (low resolution) and 300 (high resolution)
Crystal to detector distance (mm)	40
Measured and independent reflections	385562, 36109 (I>0)
(sinθ/λ) <sub>max</sub> (Å <sup>-1</sup> )	1.08
μ <sub>RX</sub> (mm <sup>-1</sup> ), A <sub>min</sub> , A <sub>max</sub>	1.6583, 0.698 and 0.805

# Electron density of Y complex

## Refinement strategy

for 9633 reflections,  $I > 3\sigma_I$

Complete Multipolar model with:

- Harmonic ADPs
- Multipolar expansion up to 4<sup>th</sup> order on Y (3<sup>rd</sup> order for other non-hydrogen atoms)
- Constrains on pyrazolyl rings (equivalent)

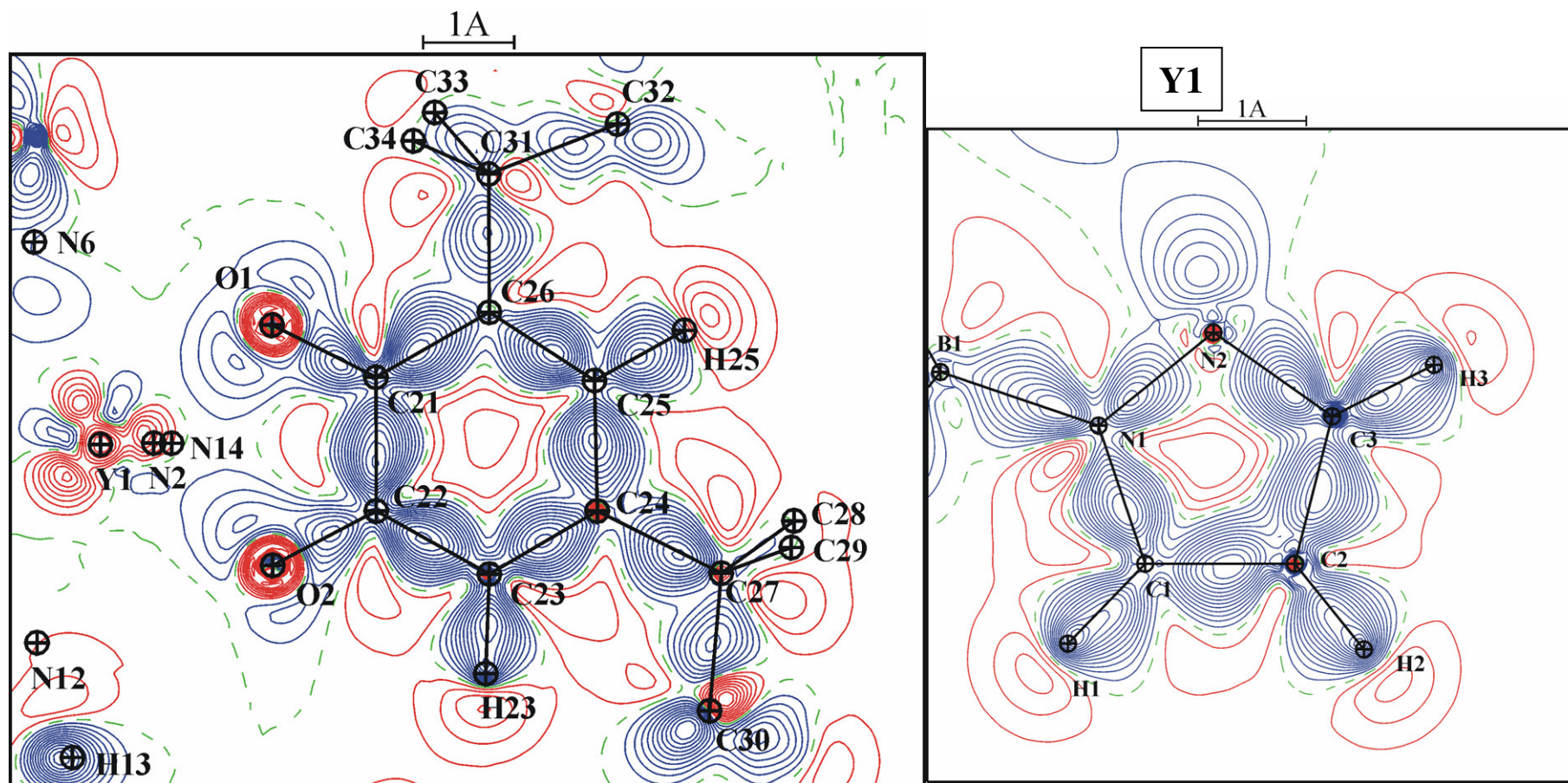
for a total of 1052 parameters

Final agreement factors

**$R = 1.47\%$ ,  $R_w = 1.47\%$ , GoF = 0.42**

# Electron density of Y complex

## Static deformation electron density

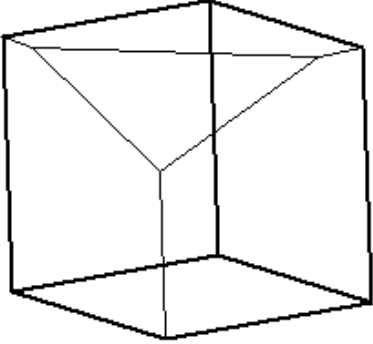
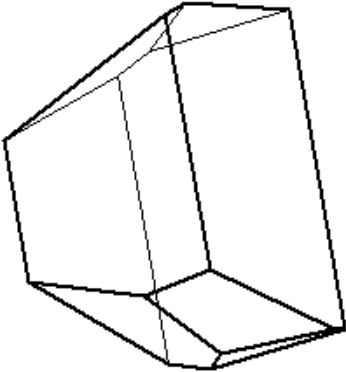


Same planes and contours as before.



# Data collection: Gd complex

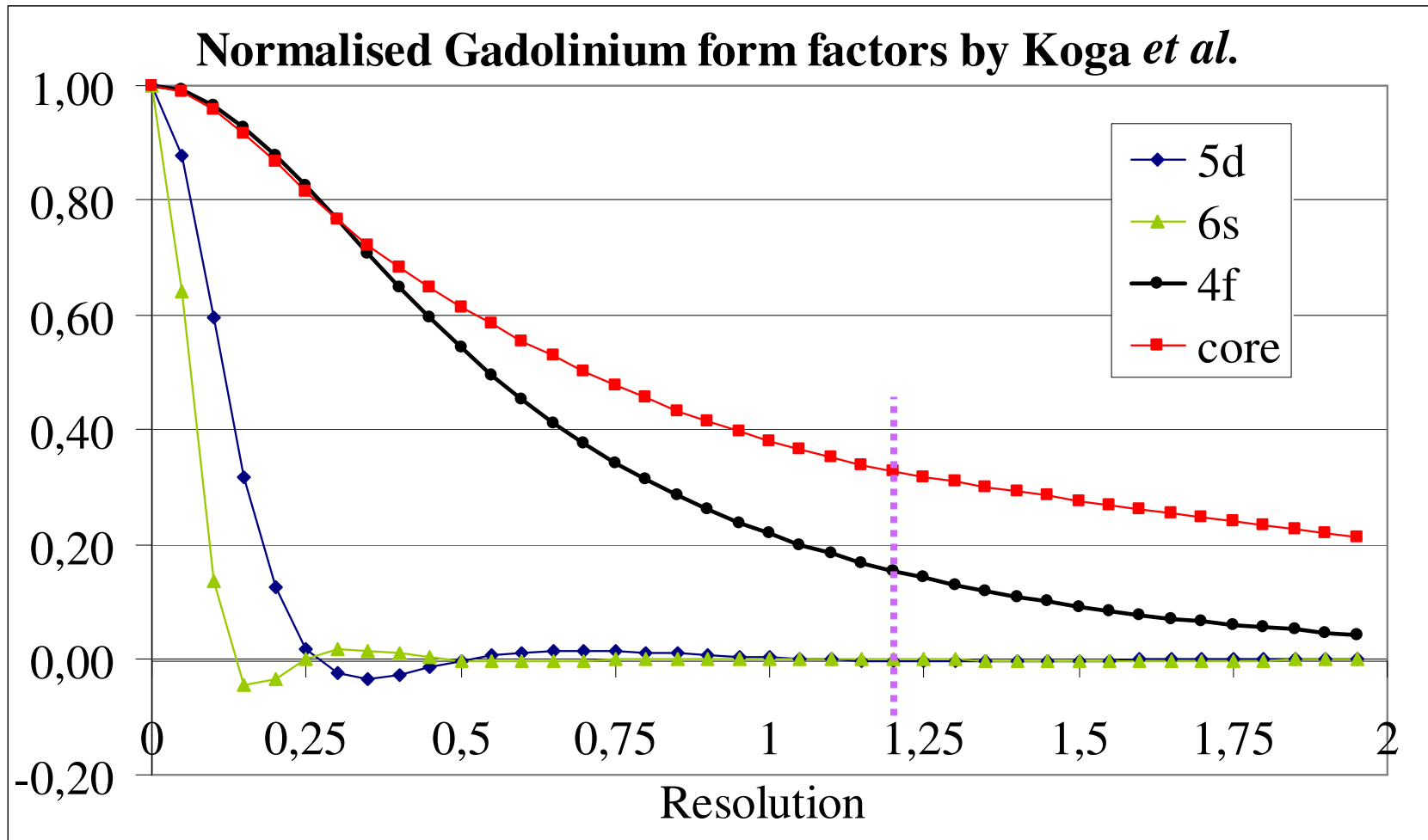
## X-ray data collection (Xcalibur)

	Crystal 1	Crystal 2
Crystal shape		
Crystal dimensions (mm)	0.15 x 0.15 x 0.18	0.14 x 0.14 x 0.20
Crystal volume (mm <sup>3</sup> )	3.953 10 <sup>-3</sup>	3.828 10 <sup>-3</sup>
Temperature (K), Wavelength (Å)	106, 0.71074 Mo(Kα)	
Indep. reflections (I>0)	49511	
Completeness (%), <Redundancy>	95.2 ,5.3	
(sinθ/λ) <sub>max</sub> (Å <sup>-1</sup> )	<b>1.20</b>	
Scale factor between crystals	<b>0.9868(7)</b>	

Absorption	$\mu_{RX}$ (mm <sup>-1</sup> )	R <sub>1</sub> (%)	R <sub>2</sub> (%)	R <sub>w</sub> (%)	GoF
Without correction	/	6.13	3.75	6.03	0.581
With correction	1.88	5.94	3.61	5.82	0.580



# Gadolinium Complex



⋮ Experimental resolution limit

# Trying Electron density modeling of the Gd complex

## Preliminary refinement using the Gd form factor from Koga *et al.*

16931 reflections,  $I > 3\sigma_I$

Main refined parameters:  
(after transfer from Y complex)

***Structural  
refinement***

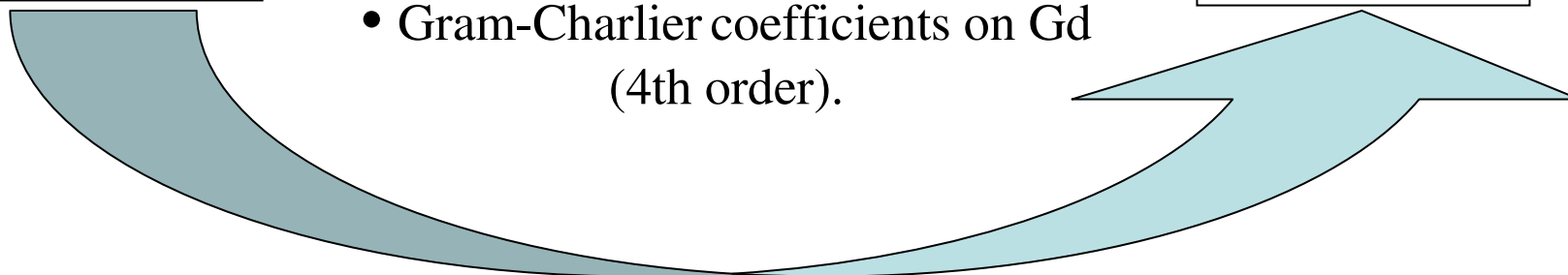
$R = 3.87 \%$ ,

$R_w = 3.41$

- Multipolar populations of organic parts,
  - Spherical model on Gd,
- Gram-Charlier coefficients on Gd  
(4th order).

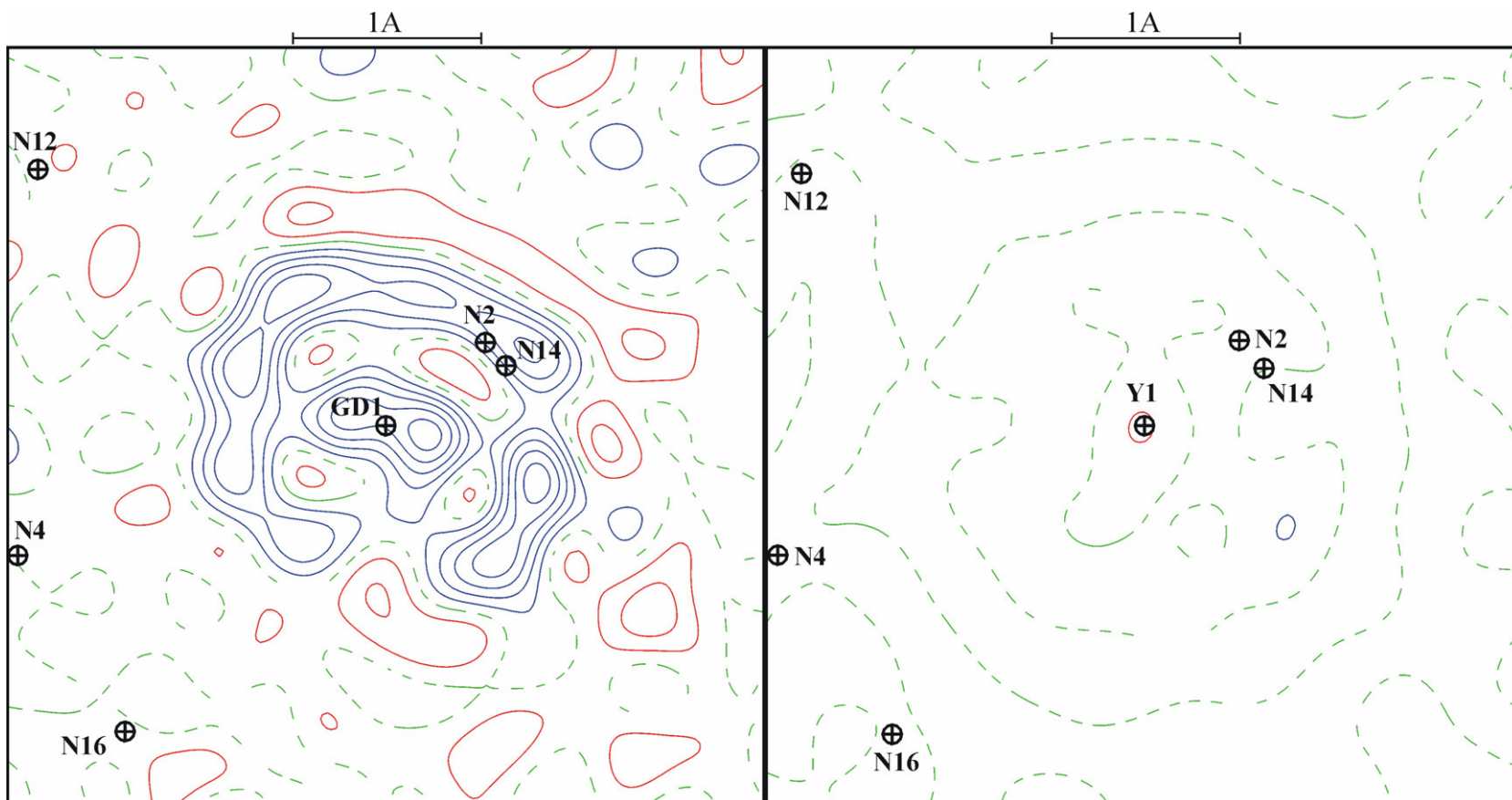
$R = 3.11 \%$

$R_w = 2.64 \%$



# Results of the preliminary refinement

## Residual electron density maps, $I > 3\sigma$



Left: Gd-O<sub>1</sub>-O<sub>2</sub> plane (form factor from Koga *et al.*), right: Y-O<sub>1</sub>-O<sub>2</sub> plane. Contours of 0.2 eÅ<sup>-3</sup>, estimated error : 1 contour.

# Available form factors

**International Tables for Crystallography**  
**Volume C: Mathematical, physical and chemical tables**

Edited by E. Prince, Department of Materials and Nuclear Engineering, University of Maryland, College Park, MD 20742, USA, and Reactor Radiation Division, National Institute of Standards and Technology, Gaithersburg, MD 20899, USA, and A. J. C. Wilson (Deceased)

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The purpose of Volume C is to provide the mathematical, physical and chemical information needed for experimental studies in structural crystallography. This new edition features two completely new chapters, on reflectometry and neutron topography. More than half of the text has been revised and updated, and there are extensive updates and corrections to tabular material.

Volume C covers all aspects of experimental techniques, using all three principal radiation types, from the selection and mounting of crystals and production of radiation through data collection and analysis to interpretation of results.

The volume is an essential source of information for all workers using crystallographic techniques in physics, chemistry, metallurgy, earth sciences and molecular biology.

<http://www.iucr.org/>

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<http://www.unb.ca/fredericton/science/chem/ajit/download.htm>

## Crystallography at SUNY Buffalo

(Best viewed at 1024 x 768 resolution)

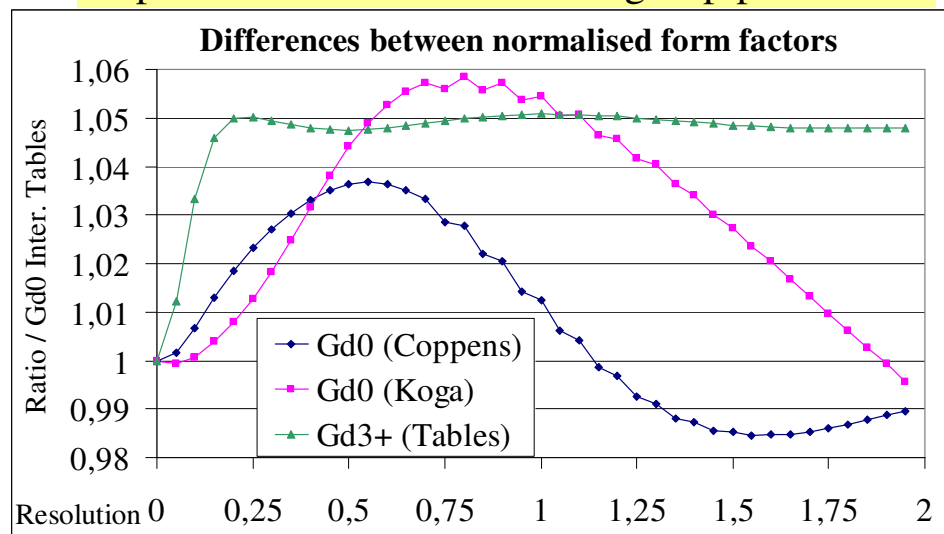
**We are in Chemical & Engineering News!**  
**Our photocrystallography work as described in Biophotonics**

- Can we combine spectroscopy and diffraction in the study of excited states of molecules in crystals?
- Can we accurately map the electron density in a crystal using X-ray diffraction, and what does this density tell us about chemical bonding?
- Can we directly measure the valence state of an atom, by analyzing the variation of its scattering power as a function of wavelength using synchrotron radiation?
- How can we maximize the accuracy of X-ray intensity data produced by area detectors with new integration procedures?

We are developing new methods to answer such questions, using equipment in our Buffalo laboratory, as well as our facilities the SUNY X3 beamline at the National Synchrotron Light Source. The following pages give information on recent publications, equipment in the laboratory and at the SUNY beamline, and on the members of our research group. For more information [call or e-mail](mailto:call_or_email) at the addresses listed.

**Metal Neuroyl Work**

<http://harker.chem.buffalo.edu/group/ptable.html>



# Do we have an accurate form factor for Gd ?

## Definition of tested models

	Model	Core definition	Population	Valence definition	Population
1	Gd <sup>3+</sup> International Tables	Cation of 61 e			
2	Gd <sup>0</sup> International Tables	Neutral atom of 64 e			
3	International Tables / Coppens <i>et al.</i>	Gd <sup>3+</sup> Inter. Tables	61 e	5d orbital calculated by Coppens <i>et al.</i>	3 e 5d <sup>3</sup>
4	International Tables / Koga <i>et al.</i>	Gd <sup>3+</sup> Inter. Tables	61 e	5d orbital calculated by Koga <i>et al.</i>	3 e 5d <sup>3</sup>
5	Coppens <i>et al.</i>	Xe type core 6s orbital included	56 e	5d and 4f orbitals	8 e 5d <sup>1</sup> , 4f <sup>7</sup>
6	Koga <i>et al.</i>	Xe type core	54 e	6s, 5d and 4f orbitals	10 e 6s <sup>2</sup> , 5d <sup>1</sup> , 4f <sup>7</sup>

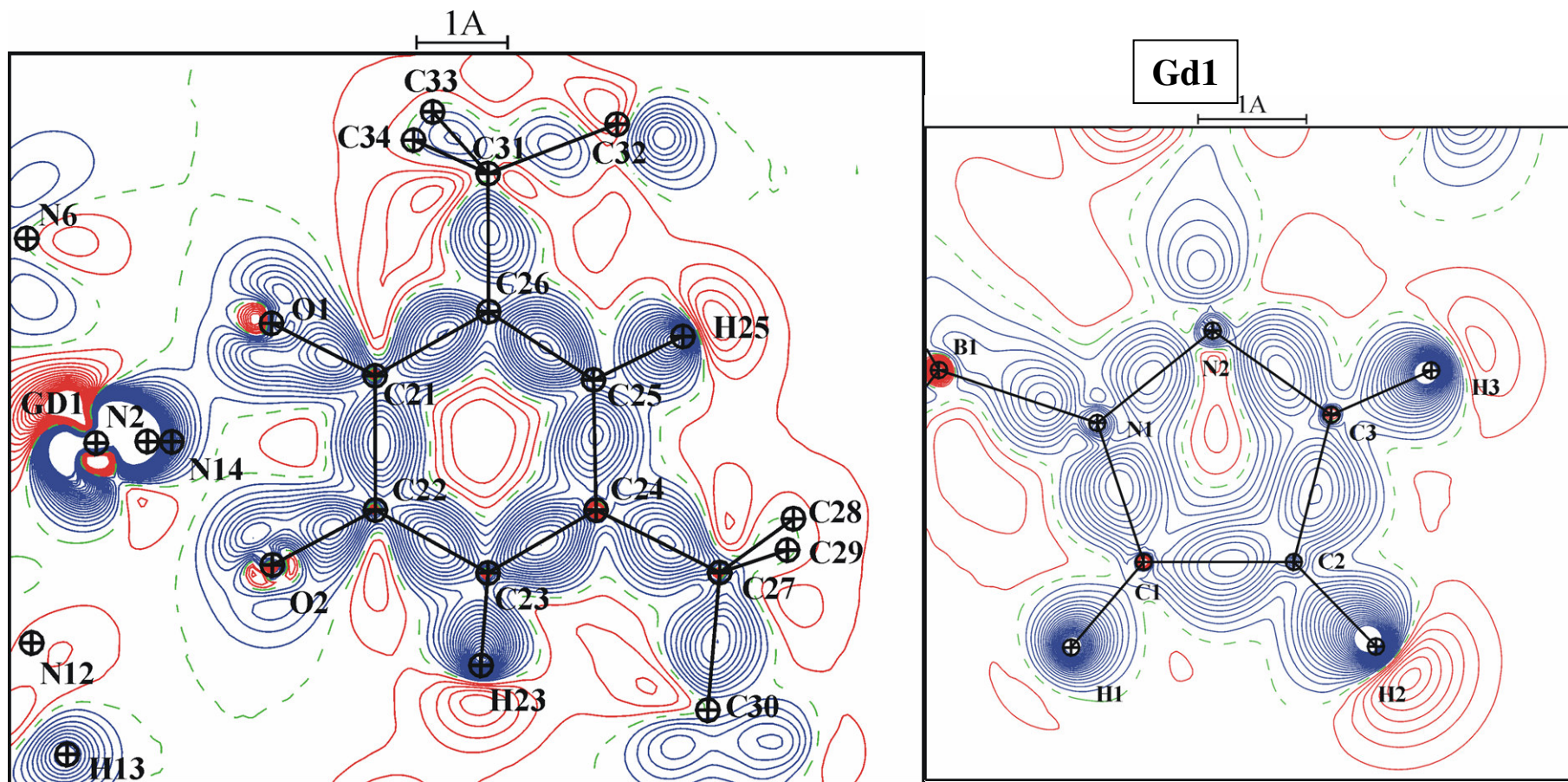
## Results for different form factor models

Model	Source	$R$ (%)	$R_w$ (%)	Scale factor variation / model 2	$U_{\text{iso}}$ Gd
<b>1</b>	Internationals Tables ( $\text{Gd}^{3+}$ )	3.68	3.22	0.00 %	0.0153
<b>2</b>	Internationals Tables ( $\text{Gd}^0$ )	3.70	3.19	/	0.0154
<b>3</b>	Internat. Tables / Coppens <i>et al.</i>	3.44	3.09	-2.69 %	0.0150
<b>4</b>	Internat. Tables / Koga <i>et al.</i>	3.60	3.04	-0.13 %	0.0153
<b>5</b>	Coppens <i>et al.</i>	4.13	3.61	-2.39 %	0.0156
<b>6</b>	Koga <i>et al.</i>	3.73	3.24	+0.09 %	0.0151



# Best multipolar refinement (model 4)

## Static deformation electron density



Static deformation electron density maps. Contours of 0.05 eÅ<sup>-3</sup>.

## **Conclusions and perspectives**

**- Increase the maximum resolution and accuracy of the data collection with synchrotron**

**- use of higher order multipoles**

**- Need of Accurate theoretical wave functions of rare earth and heavy elements which should be tested using SR data**

**- CAUTION in electron density refinement on heavy atoms which means that even with synchrotron radiation it is still frontier research.**



# More for SR diffraction...

- Ultra high resolution protein crystallography  
see B Guillot's talk
- Minerals
- Unstable crystals
- Metastable states
- Phase transitions and accurate thermal parameters
- Microcrystals
- Minerals